Problem 1: Determine the cosine or sine series (whichever is appropriate) of the following function over the given interval:

Probleem 1: *Bepaal die cosinus- of sinus-reeks (watter een ook al gepas is) van die volgende funksie oor die gegewe interval:*

$$f(x) = |\sin x|, \quad -\pi < x < \pi.$$

Problem 2: Suppose a uniform beam of length L is supported at $x = 0$ and $x = L$. It can be shown that if the load per unit length is $w(x) = w_0 x/L$, $0 < x < L$, then the differential equation for the deflection $y(x)$ is

Probleem 2: *Veronderstel 'n uniforme balk met lengte L word ondersteun by $x = 0$ en $x = L$. Daar kan getoon word dat indien die drag per eenheidslengte $w(x) = w_0 x/L$, $0 < x < L$, is, dan is die differensiaalvergelyking vir die buiging $y(x)$*

$$k\frac{d^4y}{dx^4} = \frac{w_0 x}{L},$$

where k is a constant (see p. 210 in the textbook). Expand $w(x)$ in a sine series, and use the method of undetermined coefficients to find a particular solution $y_p(x)$ of the differential equation.

met k konstant (sien bl. 210 in die handboek). Brei $w(x)$ uit as 'n sinus-reeks, en gebruik die metode van onbepaalde koëffisiënte om 'n partikuliere oplossing $y_p(x)$ vir die differensiaalvergelyking te vind.

Problem 3: Consider the function

Probleem 3: *Beskou die funksie*

$$f(x) = x(\pi^2 - x^2), \quad -\pi < x < \pi.$$

This function is odd, and can therefore be represented with a sine series. The question is: how rapidly will the coefficients in the sine series approach zero (that is, what is p in the expression $b_n \sim c/n^p$)? Do not calculate the coefficients; rather investigate the smoothness of $f(x)$.

Hierdie funksie is onewe, en kan dus met 'n sinusreeks voorgestel word. Die vraag is: hoe vinnig sal die koëffisiënte in die sinus-reeks na nul streef (d.w.s. wat is p in die uitdrukking $b_n \sim c/n^p$)? Moenie die koëffisiënte uitwerk nie; ondersoek eerder die gladheid van $f(x)$.

Problem 4: Use Parseval's theorem and the result of Problem 1 above to determine the limiting value of the following series:

Probleem 4: *Gebruik Parseval se stelling en die resultaat van Probleem 1 hierbo om die limietwaarde van die volgende reeks te bepaal:*

$$\frac{1}{1^2 \cdot 3^2} + \frac{1}{3^2 \cdot 5^2} + \frac{1}{5^2 \cdot 7^2} + \cdots.$$

Problem 5: Consider the following boundary value problem:

Probleem 5: *Beskou die volgende randwaardeprobleem:*

$$y'' + \lambda y = 0, \quad y'(0) = 0, \quad y'(L) = 0.$$

Note that the boundary conditions are specified for the derivatives of y. Show that the eigenvalues and eigenfunctions for this problem are $\lambda_n = n^2\pi^2/L^2$ and $y_n = \cos(n\pi x/L)$, $n = 0, 1, 2, \ldots$.

Let op dat die randwaardes vir die afgeleides van y gespesifiseer word. Toon aan dat die eiewaardes en eiefunksies vir hierdie probleem gegee word deur $\lambda_n = n^2\pi^2/L^2$ en $y_n = \cos(n\pi x/L)$, $n = 0, 1, 2, \ldots$.

Hint: Example 1 on p. 440 provides an outline of this derivation. Your need to fill in the details.

Wenk: Example 1 op bl. 440 voorsien 'n raamwerk van hierdie afleiding. Jy moet die nodige detail invul.