*Submit your solution to **Problem 2** on **21 September**, promptly at 14:00, in hard copy. No electronic versions will be accepted. Late submissions will be penalized severely. Cooperation is limited to the exchange of a few ideas. The exchange of code, graphs or mathematical calculations are not allowed. What you submit must be your own work.*

Jou oplossing tot **Probleem 2** moet **21 September**, stiptelik 14:00, in harde kopie ingehandig word. Geen elektroniese weergawes word aanvaar nie. Laat inhandiging sal streng gepenaliseer word. Samewerking word beperk tot die uitruil van enkele idees. Die uitruil van kode, grafieke of wiskundige berekeninge word nie toegelaat nie. Wat jy inhandig moet jou eie werk wees.

Problem 1: Suppose we are interested in the steady-state temperature in a quarter-circular plate, of which the radius is c. The temperature of the two straight edges is kept at zero and the temperature of the curved edge is kept at $f(\theta)$, $0 < \theta < \frac{\pi}{2}$. These conditions and Laplace's equation in polar coordinates lead to:

***Probleem 1:** Veronderstel ons stel belang in die gestadigde temperatuur in 'n kwartsirkel-vormige plaat, waarvan die radius c is. Die temperatuur van die twee reguit sye by nul gehou word en die temperatuur van die geboë sy by $f(\theta)$, $0 < \theta < \frac{\pi}{2}$. Hierdie voorwaardes tesame met Laplace se vergelyking in poolkoördinate lei na:*

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0, \quad u(r,0) = 0, \quad u(r,\tfrac{\pi}{2}) = 0, \quad u(c,\theta) = f(\theta).$$

Solve this problem, for $c = 1$ and

Los hierdie probleem op, vir $c = 1$ en

$$f(\theta) = \begin{cases} 100, & 0 < \theta < \frac{\pi}{4} \\ 0, & \frac{\pi}{4} < \theta < \frac{\pi}{2} \end{cases}$$

Then plot a few isotherms in the xy-plane.

Stip dan 'n klompie isoterme in the xy-vlak.

Problem 2: Consider the following function:

***Probleem 2:** Beskou die volgende funksie:*

$$f(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x \le 3 \\ 0 & x > 3 \end{cases}$$

(a) Determine the Fourier integral representation of this function. That is, find $A(\alpha)$ and $B(\alpha)$ such that

***(a)** Bepaal die Fourier-integraalvoorstelling van hierdie funksie. Dit wil sê, vind $A(\alpha)$ en $B(\alpha)$ sodat*

$$f(x) = \frac{1}{\pi}\int_0^\infty \big[A(\alpha)\cos(\alpha x) + B(\alpha)\sin(\alpha x)\big]\,d\alpha.$$

(b) Plot a number of partial integral approximations of this representation as follows. For a sufficiently large b, fix x and numerically integrate over α to approximate $f(x)$ as

***(b)** Stip 'n klompie parsiële integraalbenaderings van hierdie voorstelling, soos volg. Vir b groot genoeg, maak x vas en integreer numeries oor α om $f(x)$ te benader volgens*

$$f(x) \approx \frac{1}{\pi}\int_0^b \big[A(\alpha)\cos(\alpha x) + B(\alpha)\sin(\alpha x)\big]\,d\alpha.$$

Let x vary over say $[-2, 6]$, and calculate such an approximate value of $f(x)$ for each value of x. Generate plots for $b = 5$, 10, 25 and 50, and comment on the convergence.

Laat x oor sê $[-2, 6]$ varieër, en bereken so 'n benaderde waarde van $f(x)$ vir elke waarde van x. Genereer grafieke vir $b = 5$, 10, 25 en 50, en lewer kommentaar op die konvergensie.
