*Submit your solution to* ***Problem 2*** *by* ***14:00 on 6 September****, in hard copy. No electronic versions will be accepted. Late submissions will be penalized severely. Cooperation is limited to the exchange of a few ideas. The exchange of code, graphs or mathematical calculations are not allowed. What you submit must be your own work.*

Jou oplossing tot **Probleem 2** moet teen **6 September om 14:00** in harde kopie ingehandig word. Geen elektroniese weergawes word aanvaar nie. Laat inhandiging sal streng gepenaliseer word. Samewerking word beperk tot die uitruil van enkele idees. Die uitruil van kode, grafieke of wiskundige berekeninge word nie toegelaat nie. Wat jy inhandig moet jou eie werk wees.

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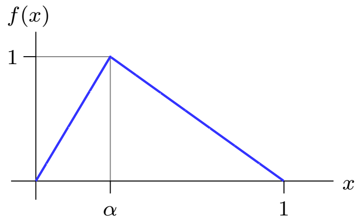
**Problem 1:** A guitar string that is plucked in the middle sounds different from one that is plucked near the end. The reason behind this lies in the harmonics (overtones). Consider a string of length $L = 1$ with initial conditions as indicated below.




$$u(x, 0) = f(x), \quad 0 < x < 1$$

$$u_t(x, 0) = 0, \quad 0 < x < 1$$

**(a)** Apply these conditions to the solution derived in Lecture 12, and write down $u(x,t)$ in terms of $\alpha$. You may use WolframAlpha to compute the necessary coefficients, but simplify the expression as far as possible.

**(b)** Where should the string be plucked so that the $n$-th harmonic is zero?

***Probleem 1:*** *'n Kitaarsnaar wat in die middel gepluk word klink anders as een wat naby 'n endpunt gepluk word. Die rede hiervoor lê in die harmonieke (ondertone). Beskou 'n snaar van lengte $L = 1$ met aanvangsvoorwaardes soos hieronder aangedui.*

***(a)*** *Pas hierdie voorwaardes toe op die oplossing wat in Lesing 12 herlei is, en skryf $u(x,t)$ in terme van $\alpha$. Jy kan WolframAlpha gebruik om die nodige koëffisiënte te bepaal, maar vereenvoudig die uitdrukking so ver moontlik.*

***(b)*** *Waar moet die snaar gepluk word sodat die $n$-de harmoniek nul is?*

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**Problem 2:** Use the result derived in Lecture 15 to write down a solution to Laplace's equation,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < 1, \quad 0 < y < 1,$$

satisfying the Dirichlet boundary conditions $u(0,y) = 0$, $u(1,y) = y$, $u(x,0) = 0$ and $u(x,1) = \sin(\frac{5\pi}{2}x)$. As usual, feel free to use WolframAlpha for the evaluation of integrals. Then generate both a surface plot and contour plot of the solution $u(x,y)$.

***Probleem 2:*** *Gebruik die resultaat in Lesing 15, en skryf 'n oplossing neer vir Laplace se vergelyking,*

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < 1, \quad 0 < y < 1,$$

*wat voldoen aan die Dirichlet randvoorwaardes $u(0,y) = 0$, $u(1,y) = y$, $u(x,0) = 0$ en $u(x,1) = \sin(\frac{5\pi}{2}x)$. Soos gewoontlik is jy welkom om WolframAlpha te gebruik vir die evaluering van integrale. Genereer dan 'n oppervlak-plot sowel as 'n kontoer-plot van die oplossing $u(x,y)$.*

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