*Submit your solution to **Problem 2** on **17 August**, promptly at 14:00, in hard copy. No electronic versions will be accepted. Late submissions will be penalized severely. Cooperation is limited to the exchange of a few ideas. The exchange of code, graphs or mathematical calculations are not allowed. What you submit must be your own work.*

Jou oplossing tot **Probleem 2** moet **17 Augustus**, stiptelik 14:00, in harde kopie ingehandig word. Geen elektroniese weergawes word aanvaar nie. Laat inhandiging sal streng gepenaliseer word. Samewerking word beperk tot die uitruil van enkele idees. Die uitruil van kode, grafieke of wiskundige berekeninge word nie toegelaat nie. Wat jy inhandig moet jou eie werk wees.

Problem 1: In Tutorial 2, Problem 3 we predicted that the coefficients in the sine series expansion of the odd function

$$f(x) = x(\pi^2 - x^2), \quad -\pi < x < \pi,$$

will approach zero like $1/n^3$.

(a) Test the validity of this prediction by calculating the coefficients explicitly (with the aid of WolframAlpha).

(b) Plot a few partial sum approximations of this sine series, and observe that convergence is indeed rather quick.

(c) It turns out that there is quite an easy alternative to find the sine series of the above function f. Consider the following equation (derived in Lecture 4):

$$x^2 = \tfrac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \tfrac{(-1)^n}{n^2} \cos nx.$$

Evaluate the indefinite integral w.r.t. x on both sides, consider $x = 0$ to solve for the integration constant, and rearrange to arrive at the sine series expansion of f.

Probleem 1: *In Tutoriaal 2, Probleem 3 het ons voorspel dat die koëffisiënte in die sinus-reeksuitbreiding van die onewe funksie na nul sal streef soos $1/n^3$.*

(a) *Toets die geldigheid van hierdie voorspelling deur die koëffisiënte eksplisiet te bepaal (met die hulp van WolframAlpha).*

(b) *Stip 'n paar parsiële sombenaderings van hierdie sinus-reeks, en let op dat konvergensie inderdaad redelik vinnig is.*

(c) *Daar is toevallig 'n alternatiewe, makliker manier om die sinus-reeks van bostaande funksie f te bepaal. Beskou die volgende vergelyking (soos in Lesing 4 herlei):*

Evalueer die onbepaalde integraal m.b.t. x aan weerskante, beskou $x = 0$ om die integrasiekonstante op te los, en herrangskik om by die sinus-reeksuitbreiding van f uit te kom.

Problem 2: Consider the following Sturm-Liouville problem:

$$y'' + \lambda y = 0, \quad y(0) + y'(0) = 0, \quad y(1) = 0.$$

(a) Determine numerically* the four smallest eigenvalues and corresponding eigenfunctions of this problem. Also plot graphs of the eigenfunctions over the interval $[0, 1]$.

*Consult appropriate graphs for rough estimates, and use a solver like `fzero` in MATLAB or `fsolve` in SciPy to refine those estimates.

(b) If the smallest eigenvalue is λ_0, and the rest are arranged such that $\lambda_0 < \lambda_1 < \lambda_2 < \ldots$, what would a good estimate for λ_{500} be?

Probleem 2: *Beskou die volgende Sturm-Liouville probleem:*

(a) *Bepaal numeries* die kleinste vier eiewaardes en ooreenstemmende eiefunksies van hierdie probleem. Stip ook grafieke van die eiefunksies oor die interval $[0, 1]$.*

**Raadpleeg toepaslike grafieke vir rowwe skattings, en gebruik 'n oplosser soos `fzero` in MATLAB of `fsolve` in SciPy om die skattings te verfyn.*

(b) *As die kleinste eiewaarde λ_0 is, en die res word rangskik sodanig dat $\lambda_0 < \lambda_1 < \lambda_2 < \ldots$, wat sal 'n goeie skatting vir λ_{500} wees?*
