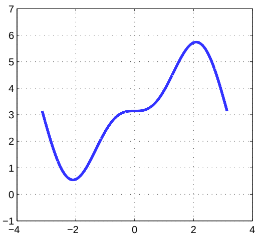
TW364 **Computer Assignment #1 Solutions** **27 July 2017**

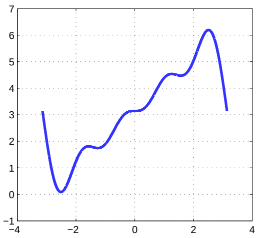
Problem 1:

(a)

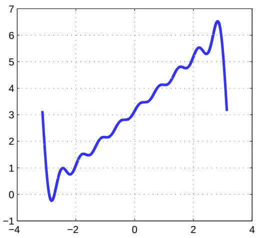
$S_3(x)$



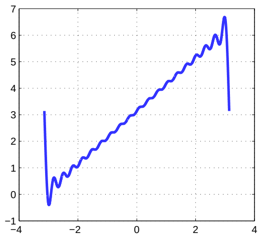
$S_5(x)$



$S_{10}(x)$

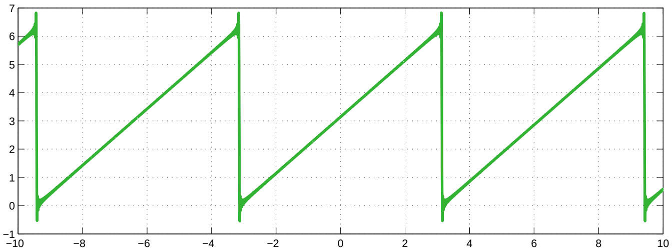


$S_{20}(x)$



(b)

$S_{200}(x)$



Gibbs oscillations are evident around the function discontinuities at $x = \pm\pi, \pm 3\pi, \ldots$.

Zooming in reveals that the overshoot is about 0.55. The discontinuities have magnitude 2π, and $(0.0895)(2\pi) = 0.562$.

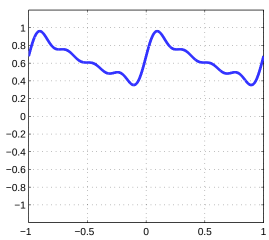
Problem 2:

(a) Making use of WolframAlpha we find the Fourier coefficients of $f(x) = \begin{cases} e^{-x-1}, & -1 < x < 0 \\ e^{-x}, & 0 \le x < 1 \end{cases}$ to be

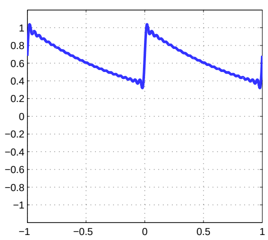
$a_0 = 2 - \frac{2}{e}$, $a_n = \frac{(e-1)(1+(-1)^n)}{e(1+n^2\pi^2)}$, $b_n = \frac{n\pi(e-1)(1+(-1)^n)}{e(1+n^2\pi^2)}$.

Hence $f(x) = 1 - \frac{1}{e} + \sum_{n=1}^{\infty} \left[\frac{(e-1)(1+(-1)^n)}{e(1+n^2\pi^2)} \cos(n\pi x) + \frac{n\pi(e-1)(1+(-1)^n)}{e(1+n^2\pi^2)} \sin(n\pi x) \right]$.

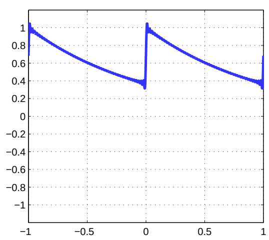
$S_{10}(x)$



$S_{50}(x)$



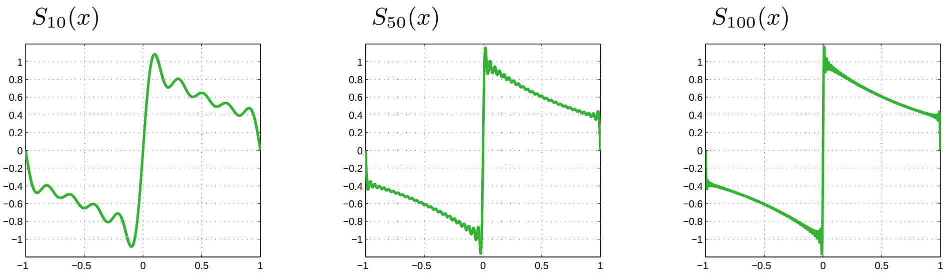
$S_{100}(x)$



(b) We find the coefficients in the sine series of the odd expansion of $f(x) = e^{-x}$ over $(-1, 1)$ to be

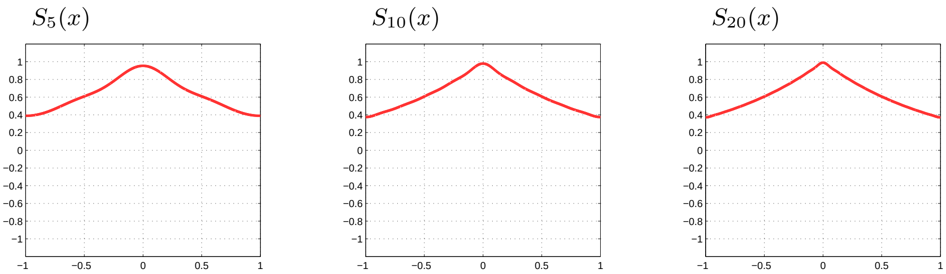
$b_n = \frac{2n\pi(e-(-1)^n)}{e(1+n^2\pi^2)}$.

Hence $f(x) = \sum_{n=1}^{\infty} \frac{2n\pi(e-(-1)^n)}{e(1+n^2\pi^2)} \sin(n\pi x)$. (*graphs on next page...*)



(c) We find the coefficients in the cosine series of the even expansion of $f(x) = e^x$ over $(-1, 1)$ to be $a_0 = 2 - \frac{2}{e}$, $a_n = \frac{2(e-(-1)^n)}{e(1+n^2\pi^2)}$.

Hence $f(x) = 1 - \frac{1}{e} + \sum_{n=1}^{\infty} \frac{2(e-(-1)^n)}{e(1+n^2\pi^2)} \cos(n\pi x)$.



(d) The approximations we get with the cosine series in part (c) are quite clearly better than the other two, in the sense that they converge to $f(x)$ much quicker and exhibit no Gibbs oscillations. The even expansion of $f(x)$ contains no discontinuities (which implies no Gibbs oscillations) and the Fourier coefficients decrease at a rate $1/n^2$ (much quicker than the $1/n$ rate of the other two).

Problem 3:

Plots of a few partial sum approximations, like the one on the right, indicate that the function f is likely

$$f(x) = \begin{cases} 1 - x, & 0 < x < 1 \\ 0, & 1 \leq x < 2 \end{cases}$$

and the series is a cosine series of the even expansion of this function.