TW 314 (Toegepaste Diskrete Wiskunde)

Tutoriaal 9: 6 April 2017

(Oplossings)

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1. (a) 00000 10101 01110 11011

   (b) $n = 4; k = 2; M = 4; d = 3$.

   (c) een fout.

   (d) nee.

   (e) Die dekoderingstabel is:

00000	10101	01110	11011
10000	00101	11110	01011
01000	11101	00110	10011
00100	10001	01010	11111
00010	10111	01100	11001
00001	10100	01111	11010
11000	01101	10110	00011
10010	00111	11100	01001

   (f) i. fout is 00010, dus kodewoord is $01100 - 00010 = 01110$.

   ii. ...meer as een fout.

   iii. fout is 00000, dus kodewoord is $01110 - 00000 = 01110$.

   iv. fout is 00000, dus kodewoord is $11011 - 00000 = 11011$.

   (g) $H = \begin{bmatrix} 11100 \\ 01010 \\ 10001 \end{bmatrix}$

   (h) $x_1 + x_2 + x_3 = 0$
   $x_2 + x_4 = 0$
   $x_1 + x_5 = 0$

   (i) $GH^T = \begin{bmatrix} 000 \\ 000 \end{bmatrix}$

   (j)

error	syndrome
10000	101
01000	110
00100	100
00010	010
00001	001

   (k) does not appear in look-up table...more than one error.

2. (a) 0000 0112 0221 1011 1120 1202 2022 2101 2210

   (b) $n = 5; k = 2; M = 9; d = 3$.

   (c) een fout.

   (d) ja.

(e) Die dekoderingstabel is:

0000	0112	0221	1011	1120	1202	2022	2101	2210
1000	1112	1221	2011	2120	2202	0022	0101	0210
2000	2112	2221	0011	0120	0202	1022	1101	1210
0100	0212	0021	1111	1220	1002	2122	2201	2010
0200	0012	0121	1211	1020	1102	2222	2001	2110
0010	0122	0201	1021	1100	1212	2002	2111	2220
0020	0102	0211	1001	1110	1222	2012	2121	2200
0001	0110	0222	1012	1121	1200	2020	2102	2211
0002	0111	0220	1010	1122	1201	2021	2100	2212

(f) i. fout is 0020, dus kodewoord is $2121 - 0020 = 2101$.
ii. fout is 0002, dus kodewoord is $1201 - 0002 = 1202$.
iii. fout is 0200, dus kodewoord is $2222 - 0200 = 2022$.
iv. fout is 0020, dus kodewoord is $1110 - 0020 = 1120$.

(g) $H = \begin{bmatrix} 2210 \\ 2101 \end{bmatrix}$

(h) $2x_1 + 2x_2 + x_3 = 0$
$2x_1 + x_2 + x_4 = 0$

(i) $GH^T = \begin{bmatrix} 00 \\ 00 \end{bmatrix}$

(j)

error	syndrome
1000	22
2000	11
0100	21
0200	12
0010	10
0020	20
0001	01
0002	02

(k) $S(\underline{y}) = 12$; so $\underline{e} = 0200$; so kodewoord is $\underline{x} = \underline{y} - \underline{e} = 0221$.

3. (a) $S(\mathbf{y}) = (\ \sum_{i=1}^{10} y_i\ ,\ \sum_{i=1}^{10} iy_i\ )$.

(b) $S(\mathbf{y}) = (\ (\sum_{i=1}^{10} x_i) + k\ ,\ (\sum_{i=1}^{10} ix_i) + jk\ ) = (k, jk)$.

(c) $S(0617960587) = (5, 9)$, so $k = 5$ and $j = 4$ and therefore codeword is $0617960587 - 0005000000 = 0612960587$.
$S(3617960587) = (8, 1)$, so $k = 8$ and $j = 7$ and therefore codeword is $3617960587 - 0000008000 = 3617963587$.

TW 314 (Applied Discrete Mathematics)

Tutorial 9: 6 April 2017

(Solutions)

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1. (a) 00000 10101 01110 11011

   (b) $n = 5; k = 2; M = 4; d = 3$.

   (c) one error.

   (d) no.

   (e) The decoding table is:

00000	10101	01110	11011
10000	00101	11110	01011
01000	11101	00110	10011
00100	10001	01010	11111
00010	10111	01100	11001
00001	10100	01111	11010
11000	01101	10110	00011
10010	00111	11100	01001

   (f) i. error is 00010, hence codeword is $01100 - 00010 = 01110$.

   ii. ...more than one error.

   iii. error is 00000, hence codeword is $01110 - 00000 = 01110$.

   iv. error is 00000, hence codeword is $11011 - 00000 = 11011$.

   (g) $H = \begin{bmatrix} 11100 \\ 01010 \\ 10001 \end{bmatrix}$

   (h) $x_1 + x_2 + x_3 = 0$
   $x_2 + x_4 = 0$
   $x_1 + x_5 = 0$

   (i) $GH^T = \begin{bmatrix} 000 \\ 000 \end{bmatrix}$

   (j)

error	syndrome
10000	101
01000	110
00100	100
00010	010
00001	001

   (k) does not appear in look-up table...more than one error.

2. (a) 0000 0112 0221 1011 1120 1202 2022 2101 2210

   (b) $n = 4; k = 2; M = 9; d = 3$.

   (c) one error.

   (d) yes.

(e) The decoding table is:

0000	0112	0221	1011	1120	1202	2022	2101	2210
1000	1112	1221	2011	2120	2202	0022	0101	0210
2000	2112	2221	0011	0120	0202	1022	1101	1210
0100	0212	0021	1111	1220	1002	2122	2201	2010
0200	0012	0121	1211	1020	1102	2222	2001	2110
0010	0122	0201	1021	1100	1212	2002	2111	2220
0020	0102	0211	1001	1110	1222	2012	2121	2200
0001	0110	0222	1012	1121	1200	2020	2102	2211
0002	0111	0220	1010	1122	1201	2021	2100	2212

(f) i. error is 0020, hence codeword is $2121 - 0020 = 2101$.
ii. error is 0002, hence codeword is $1201 - 0002 = 1202$.
iii. error is 0200, hence codeword is $2222 - 0200 = 2022$.
iv. error is 0020, hence codeword is $1110 - 0020 = 1120$.

(g) $H = \begin{bmatrix} 2210 \\ 2101 \end{bmatrix}$

(h) $2x_1 + 2x_2 + x_3 = 0$
$2x_1 + x_2 + x_4 = 0$

(i) $GH^T = \begin{bmatrix} 00 \\ 00 \end{bmatrix}$

(j)

error	syndrome
1000	22
2000	11
0100	21
0200	12
0010	10
0020	20
0001	01
0002	02

(k) $S(\underline{y}) = 12$; so $\underline{e} = 0200$; so codeword is $\underline{x} = \underline{y} - \underline{e} = 0221$.

3. (a) $S(\mathbf{y}) = (\ \sum_{i=1}^{10} y_i\ ,\ \sum_{i=1}^{10} iy_i\ )$.

(b) $S(\mathbf{y}) = (\ (\sum_{i=1}^{10} x_i) + k\ ,\ (\sum_{i=1}^{10} ix_i) + jk\ ) = (k, jk)$.

(c) $S(0617960587) = (5, 9)$, so $k = 5$ and $j = 4$ and therefore codeword is $0617960587 - 0005000000 = 0612960587$.
$S(3617960587) = (8, 1)$, so $k = 8$ and $j = 7$ and therefore codeword is $3617960587 - 0000008000 = 3617963587$.