TW 314 (Toegepaste Diskrete Wiskunde)

Tutoriaal 8: 30 Maart 2017

(Oplossings)

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1. (a) Elementêre ry-omvormings lewer $\begin{array}{|c|}\hline 10010\\ 11011\\ 10110\\ \hline\end{array} \rightarrow \begin{array}{|c|}\hline 10010\\ 01001\\ 00100\\ \hline\end{array}$

   'n Basis is dus $\{(1,0,0,1,0)(0,1,0,0,1),(0,0,1,0,0)\}$.

   (b) $dim(C) = 3$

   (c) $|C| = 2^3 = 8$

   (d) $(0,0,0,0,0)$
   $(0,0,1,0,0)$
   $(0,1,0,0,1)$
   $(0,1,1,0,1)$
   $(1,0,0,1,0)$
   $(1,0,1,1,0)$
   $(1,1,0,1,1)$
   $(1,1,1,1,1)$

2. (a) Elementêre ry-omvormings lewer

$$\begin{array}{|c|}\hline 5223\\ 3412\\ 2204\\ 6045\\ \hline\end{array} \rightarrow \begin{array}{|c|}\hline 1662\\ 1653\\ 1102\\ 1032\\ \hline\end{array} \rightarrow \begin{array}{|c|}\hline 1662\\ 0061\\ 0210\\ 0140\\ \hline\end{array} \rightarrow \begin{array}{|c|}\hline 1662\\ 0140\\ 0061\\ 0210\\ \hline\end{array} \rightarrow \begin{array}{|c|}\hline 1662\\ 0140\\ 0061\\ 0000\\ \hline\end{array} \rightarrow \begin{array}{|c|}\hline 1032\\ 0140\\ 0016\\ \hline\end{array} \rightarrow \begin{array}{|c|}\hline 1005\\ 0104\\ 0016\\ \hline\end{array}$$

   'n Basis is dus $\{(1,0,0,5)(0,1,0,4),(0,0,1,6)\}$.

   (b) $dim(C) = 3$

   (c) $|C| = 7^3$

3. (a) Die som van twee vektore van C is weer in C, en alle veelvoude van vektore van C is in C. Dus volg van Stelling 2.18 dat C 'n deelruimte is van $V(3,5)$.

   Of, die linêer onafhanklike versameling $\{(3,4,1)\}$ bring al die vektore van C voor, d.i. C bestaan uit die vyf veelvoude (oor $GF(5)$)van $(3,4,1)$. Dus C 'n deelruimte van $V(3,5)$ met basis $\{(3,4,1)\}$.

   (b) Pas die permutasies (1243) en (1342) toe op kolomme 2 en 3 onderskeidelik:

$$\begin{array}{|c|}\hline 000\\ 132\\ 214\\ 341\\ 423\\ \hline\end{array} \rightarrow \begin{array}{|c|}\hline 000\\ 112\\ 224\\ 331\\ 443\\ \hline\end{array} \rightarrow \begin{array}{|c|}\hline 000\\ 111\\ 222\\ 333\\ 444\\ \hline\end{array}$$

   Dus C is ekwivalent aan die 5-êre repetisiekode van lengte 3.

TW 314 (Applied Discrete Mathematics)

Tutorial 8: 30 March 2017

(Solutions)

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1. (a) Elementary row operations give $\boxed{\begin{matrix}10010\\11011\\10110\end{matrix}} \rightarrow \boxed{\begin{matrix}10010\\01001\\00100\end{matrix}}$

   A basis is therefore $\{(1,0,0,1,0)(0,1,0,0,1),(0,0,1,0,0)\}$.

   (b) $dim(C) = 3$

   (c) $|C| = 2^3 = 8$

   (d) $(0,0,0,0,0)$
   $(0,0,1,0,0)$
   $(0,1,0,0,1)$
   $(0,1,1,0,1)$
   $(1,0,0,1,0)$
   $(1,0,1,1,0)$
   $(1,1,0,1,1)$
   $(1,1,1,1,1)$

2. (a) Elementary row operations give

   $$\boxed{\begin{matrix}5223\\3412\\2204\\6045\end{matrix}} \rightarrow \boxed{\begin{matrix}1662\\1653\\1102\\1032\end{matrix}} \rightarrow \boxed{\begin{matrix}1662\\0061\\0210\\0140\end{matrix}} \rightarrow \boxed{\begin{matrix}1662\\0140\\0061\\0210\end{matrix}} \rightarrow \boxed{\begin{matrix}1662\\0140\\0061\\0000\end{matrix}} \rightarrow \boxed{\begin{matrix}1032\\0140\\0016\end{matrix}} \rightarrow \boxed{\begin{matrix}1005\\0104\\0016\end{matrix}}$$

   A basis is therefore $\{(1,0,0,5)(0,1,0,4),(0,0,1,6)\}$.

   (b) $dim(C) = 3$

   (c) $|C| = 7^3$

3. (a) The sum of two vectors of C is again in C, and all multiples of vectors of C is in C. It therefore follows from Theorem 2.18 that C is a subspace of $V(3,5)$.

   Or, the linearly independent set $\{(3,4,1)\}$ generates all the vectors of C, *i.e.* C consists of the five multiples (over $GF(5)$)of $(3,4,1)$. Thus C is a subspace of $V(3,5)$ with basis $\{(3,4,1)\}$.

   (b) Apply the permutations (1243) and (1342) on columns 2 and 3 respectively:

   $$\boxed{\begin{matrix}000\\132\\214\\341\\423\end{matrix}} \rightarrow \boxed{\begin{matrix}000\\112\\224\\331\\443\end{matrix}} \rightarrow \boxed{\begin{matrix}000\\111\\222\\333\\444\end{matrix}}$$

   Therefore C is equivalent to the 5-ary repetition code of length 3.
