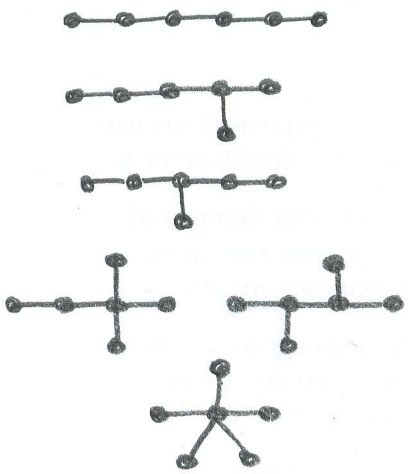
Tutorial 5 (Solutions)

#1.



#2. Let x be the # leaves.

Then $1 \cdot x + 2 \cdot 3 + 5 \cdot (10 - x) = 2 \cdot 12$

and therefore $x = 8$; so

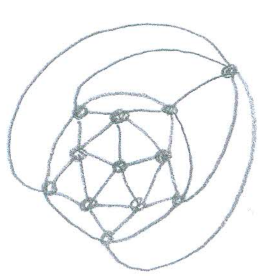
there are 8 leaves.

#3. G:

This drawing shows that G is planar.

The graph H has order $p = 6$ and size $q = 13$. Since $q > 3p - 6$, H is not planar.

#4.



This is the graph of the icosahedron.

#5. Every planar graph has a vertex of degree at most five.

Proof: Let G be a planar graph of order p and size q. If $\deg(v) \geq 6$ for all $v \in V(G)$, then $2q = \sum_{v \in V(G)} \deg(v) \geq 6p$ and therefore $q \geq 3p$. This contradicts $q \leq 3p - 6$. Therefore at least one vertex of G must have degree at most five.

#6. Suppose G is a planar graph of order p and size q, and that every vertex of G has degree at least five. Then $2q = \sum_{v \in V(G)} \deg(v) \geq 5p$. Since $q \leq 3p - 6$, it follows that $5p \leq 6p - 12$, and therefore $p \geq 12$. Therefore, if G has fewer than 12 vertices, then it must have a vertex of degree at most four.

#7. $d = 2$: K_3, or the triangle

$d = 3$: K_4, or the tetrahedron

$d = 4$: the octahedron

$d = 5$: the icosahedron

#8. There exists only one 4-regular maximal planar graph

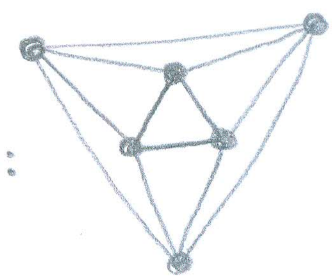
<u>Proof</u>: Let G be a 4-regular maximal planar graph of order p and size q.

Then $q = 3p - 6$, and therefore

$$4p = \sum_{v \in V(G)} \deg(v) = 6p - 12;$$ hence $p = 6$ and $q = 12$.

A 4-regular maximal planar graph of order 6 and size 12 is given by the graph G — which is the graph of the octahedron.

G:



(In fact, by using the Havel-Hakimi Theorem, you can show that, up to isomorphism, G is the only graph with degree sequence 4, 4, 4, 4, 4, 4.)