Tutorial 4 Solutions

1.

	order	size	δ	Δ
(a)	n	$\binom{n}{2}$	$n-1$	$n-1$
(b)	$r+s$	rs	$\min\{r,s\}$	$\max\{r,s\}$
(c)	n	0	0	0
(d)	$r+s$	$\binom{r}{2}+\binom{s}{2}$	$\min\{r-1,s-1\}$	$\max\{r-1,s-1\}$

2. Let x be the number of vertices of degree 3 and y the number of vertices of degree 5.

Then $\sum_{v \in V} \deg(v) = |2E| \Rightarrow 3x+5y = 50$

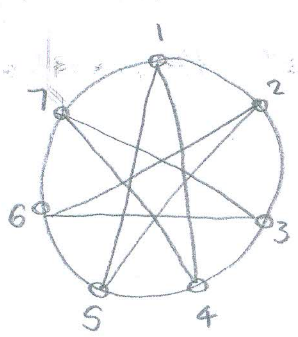
and $x+y = 14$. Solving gives $x = 10$.

Therefore 10 vertices have degree 3.

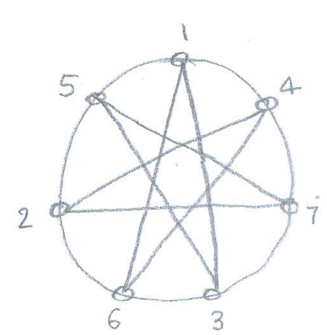
3. The set $\{1,2,3,4,5\}$ of vertices of H induces a 5-cycle (that is, a C_5). G contains no such 5-cycle, and therefore $G \not\cong H$.

4.

G:



H:



From this redrawing of H we can write down an isomorphism $\phi: V(G) \longrightarrow V(H)$:

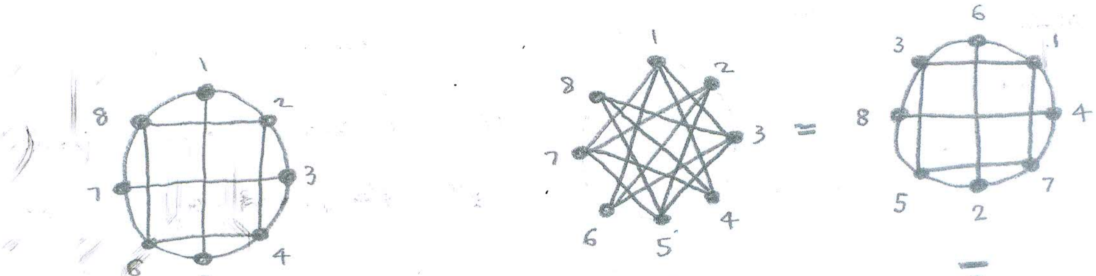
$\phi(1)=1$, $\phi(2)=4$, $\phi(3)=7$, $\phi(4)=3$, $\phi(5)=6$, $\phi(6)=2$, $\phi(7)=5$.

This shows that $G \cong H$.

Or reason as follows: Both $\bar{G}$ and $\bar{H}$ are 5-cycles (that is, a C_5) —— to see this, draw the graphs $\bar{G}$ and $\bar{H}$. Therefore $\bar{G} \cong \bar{H}$, and therefore $G \cong H$.

5. Suppose G is self-complementary of order n. Then, since $G \cong \bar{G}$, both these graphs must have size $\frac{1}{2}\binom{n}{2} = \frac{n(n-1)}{4}$. It follows that n or $n-1$ is divisible by 4, that is, $n \equiv 0$ or $1 \pmod 4$.

6.



G $\qquad$ $\bar{G}$ $\qquad$ $\bar{G}$.

An isomorphism $\phi: V(G) \to V(\bar{G})$ is

$\phi(1)=6$, $\phi(2)=1$, $\phi(3)=4$, $\phi(4)=7$,

$\phi(5)=2$, $\phi(6)=5$, $\phi(7)=8$, $\phi(8)=3$.

Therefore $G \cong \bar{G}$, and therefore G is self-complementary.

7.