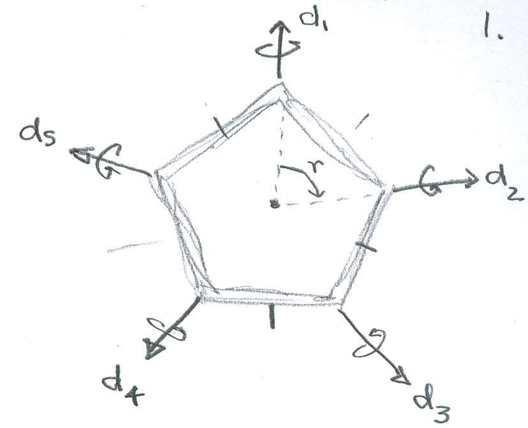
Tutorial 3 (Solutions)



1. $G = \{ id, r, r^2, r^4, r^5, d_1, d_2, d_3, d_4, d_5 \}$

and $|X| = 2^5$.

(a)

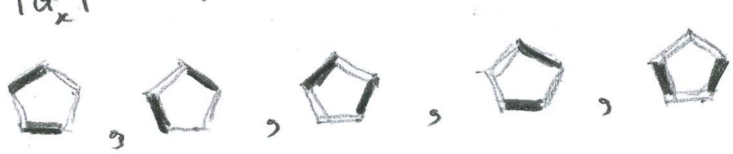
$g \in G$	$\lvert X_g \rvert$
id	2^5
r, r^2, r^3, r^4	2^1
d_1, d_2, d_3, d_4, d_5	2^3

$$\sum_{g \in G} |X_g| = 2^5 + 4 \times 2^1 + 5 \times 2^3 = 80.$$

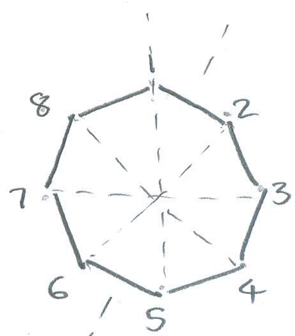
Therefore # orbits $= \dfrac{1}{|G|} \displaystyle\sum_{g \in G} |X_g| = \dfrac{80}{10} = 8.$

(b) $G_x = \{ id, d_2 \}$. Therefore $|G_x| = 2$ and

$|Gx| = \dfrac{|G|}{|G_x|} = \dfrac{10}{2} = 5$. In fact,

$Gx = \{$  $\}$

2. Let the group G of symmetries of the regular octagon act on the set X of all colourings of its corners, 3 corners are to be black and 5 white.



G has the elements

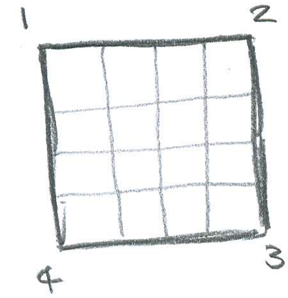
id | $s_1 = (12)(38)(47)(56)$
$\rho = (12345678)$ | $s_2 = (23)(14)(85)(76)$
$\rho^2 = (1357)(2468)$ | $s_3 = (34)(25)(16)(87)$
$\rho^3 = (14725836)$ | $s_4 = (45)(36)(27)(18)$
$\rho^4 = (15)(26)(37)(48)$ | $t_1 = (28)(37)(46)$
$\rho^5 = (16385274)$ | $t_2 = (13)(84)(75)$
$\rho^6 = (1753)(2864)$ | $t_3 = (24)(15)(86)$
$\rho^7 = (18765432)$ | $t_4 = (35)(26)(17)$

$g \in G$	$\lvert X_g \rvert$
id	56
$\rho, \rho^3, \rho^5, \rho^7$	0
ρ^2, ρ^6	0
ρ^4	0
s_1, s_2, s_3, s_4	0
t_1, t_2, t_3, t_4	6

$$\sum_{g \in G} \lvert X_g \rvert = 56 + 4 \times 6 = 80$$

Therefore # necklaces $= \frac{80}{16} = 5$

3. Let the group G of symmetries of the square act on the set X of $\binom{16}{2}$ configurations.



G has the elements

id	$s_1 = (12)(34)$
$r = (1234)$	$s_2 = (23)(14)$
$r^2 = (13)(24)$	$t_1 = (13)$
$r^3 = (1432)$	$t_2 = (24)$.

$g \in G$	$\lvert X_g \rvert$
id	120
r, r^3	0
r^2	8
s_1, s_2	8
t_1, t_2	$\binom{4}{2} + 6 = 12$

Therefore # id cards

$$= \# \text{ orbits} = \frac{1}{|G|} \sum_{g \in G} |X_g| = \frac{168}{8} = 21$$