

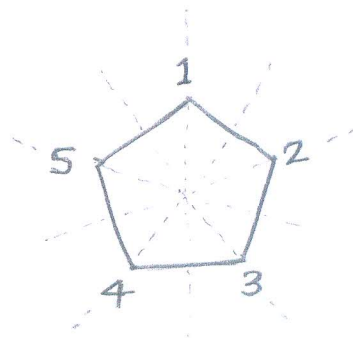
## Tutorial 2 Solutions

1. (i) No, it does not contain the identity.  
(ii) Yes, it is closed under composition of permutations. In fact, it is the subgroup  $\langle (12345) \rangle$  of  $S_5$ .  
(iii) Yes, it is closed under composition of permutations.  
(iv) No, the number of elements does not divide  $5!$ .

2. (i) The order is  $\text{lcm}(4, 2, 2) = 4$ .  
(ii) The order is  $\text{lcm}(2, 2, 2, 2) = 2$ .  
(iii) The order is  $\text{lcm}(5, 3) = 15$ .

3. The symmetries are

|           |            |
|-----------|------------|
| id        | $(23)(14)$ |
| $(12345)$ | $(34)(25)$ |
| $(13524)$ | $(45)(13)$ |
| $(14253)$ | $(15)(24)$ |
| $(15432)$ | $(12)(35)$ |



4. (a)  $\text{id}$   $m_1 = (14)(25)(34)$   
 $r = (16)(34)(29)$   $m_2 = (13)(46)$

(b)  $G_1 = G_3 = G_4 = G_6 = \{1, 3, 4, 6\}$   
 $G_2 = G_5 = \{2, 5\}$

There are two orbits.

(c)  $G_1 = G_3 = G_4 = G_6 = \{\text{id}\}$   
 $G_2 = G_5 = \{\text{id}, m_2\}$

(d)  $|G_1||G_1| = 4 \times 1 = 4$  — similar for 3, 4, 6.  
 $|G_2||G_2| = 2 \times 2 = 4$  — similar for 5.

5. (a)  $\text{id}$   $m_1 = (13)(24)$   
 $r = (14)(23)(57)$   $m_2 = (12)(34)(57)$

(b)  $G_1 = G_2 = G_3 = G_4 = \{1, 2, 3, 4\}$   
 $G_5 = G_7 = \{5, 7\}$

$G_6 = \{6\}$  — there are three orbits.

(c)  $G_1 = G_2 = G_3 = G_4 = \{\text{id}\}$   
 $G_5 = G_7 = \{\text{id}, m_1\}$   
 $G_6 = G$

(d)  $|G_1||G_1| = 4 \times 1 = 4$  — similar for 2, 3, 4  
 $|G_5||G_5| = 2 \times 2 = 4$  — similar for 5  
 $|G_6||G_6| = 1 \times 4 = 4$

6(a) id  $m_1 = (12)(84)(75)(69)$   
 $r_1 = (369)(258)(147)$   $m_2 = (45)(27)(18)(39)$   
 $r_2 = (396)(285)(174)$   $m_3 = (78)(15)(24)(36)$

(b)  $G_1 = G_2 = G_4 = G_5 = G_7 = G_8 = \{1, 2, 4, 5, 7, 8\}$   
 $G_3 = G_6 = G_9 = \{3, 6, 9\}$  — two orbits

(c)  $G_1 = G_2 = G_4 = G_5 = G_7 = G_8 = \{id\}$   
 $G_3 = \{id, m_1\}$ ,  $G_6 = \{id, m_2\}$ ,  $G_9 = \{id, m_3\}$   
 — two orbits

(d)  $|G_1| |G_1| = 6 \times 1 = 6$  — similar for 2, 4, 5, 7, 8  
 $|G_3| |G_3| = 3 \times 2 = 6$  — similar for 6, 9.