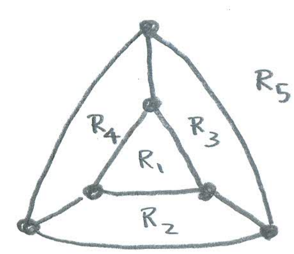
THE DUAL OF A PLANE GRAPH

Let G be a plane graph of order p and size q. Assume that $\delta(G) \geq 3$.

For a region R of G, we define the degree $\deg(R)$ of R to be the number of edges on the boundary of R. Since $\delta(G) \geq 3$, we find that all the regions are bounded by cycles, so that a region of degree r is bounded by an r-cycle.

Example: The plane graph G has $\deg(R_1) = \deg(R_5) = 3$ and $\deg(R_2) = \deg(R_3) = \deg(R_4) = 4$.

G:

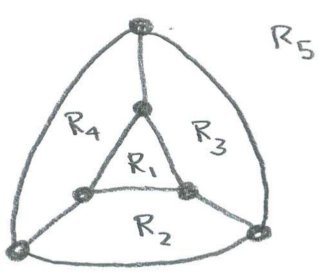


For a plane graph G with $\delta(G) \geq 3$, we construct the plane graph G^* as follows. Corresponding to each region R of G, there is a vertex R^* of G^*, and corresponding to each edge e of G, there is an edge e^* of G;

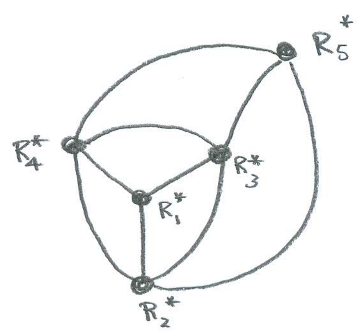
Two vertices R_1^* and R_2^* are joined by the edge e^* in G^* if and only if there corresponding regions R_1 and R_2 in G are separated by the corresponding edge e in G. The graph G^* is called the <u>dual</u> of the graph G.

<u>Example</u> (of a graph G and its dual G^*)

G:



G^*:



Note that, since we assume that the plane graph G has $\delta(G) \geq 3$, its dual G^* is a well-defined plane graph with $\delta(G^*) \geq 3$, and that $(G^*)^* = G$, that is, the dual of the dual of G is again G. Also, note that a region of degree r in G corresponds to a vertex of degree r in G^*, and that a vertex of degree p in G corresponds to a region of degree p in G^*, and that G and G^* has equal sizes.