

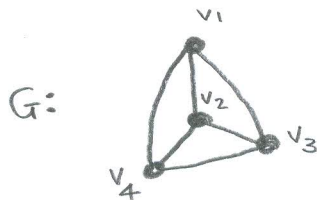
PLANAR GRAPHS

A graph that can be drawn in the plane with no of its edges crossing is called a planar graph, and such a drawing is then called an embedding of the graph.

A plane graph is an embedding (in the plane) of a planar graph.

Example: Let G be the complete graph on the four vertices v_1, v_2, v_3, v_4 .

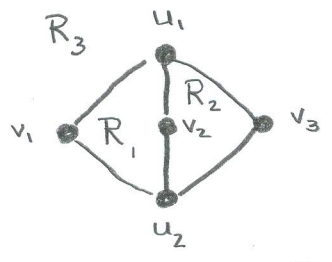
Then G is a planar graph — an embedding of G is given by



Embedding a graph in the plane is equivalent to embedding it on a sphere — convince yourself of this by considering a stereographic projection of the sphere onto the plane.

Let G be a plane graph. The connected open subsets of the plane that remains when the vertices and edges are removed are called the regions of G . Every plane graph G has exactly one unbounded region, called the exterior region of G .

Example: Let G be the complete bipartite graph with partite sets $V_1 = \{u_1, u_2\}$ and $V_2 = \{v_1, v_2, v_3\}$. An embedding of G is



It has three regions R_1, R_2 and R_3 . R_3 is the exterior region.

Theorem 6 Euler's Theorem

If G is a connected plane graph with p vertices and q edges and r regions, then $p+r = q+2$.

