CONNECTED GRAPHS AND TREES

Definitions: In a graph G with vertices u and v, a u-v path is a sequence $v_0 v_1 \dots v_n$ of distinct vertices with $v_0 = u$ and $v_n = v$ and such that G contains the edges $v_0v_1, v_1v_2, \dots, v_{n-1}v_n$. The number n of edges is the length of the u-v path.

A cycle in a graph G is a sequence $v_1 v_2 \dots v_n v_1$ of distinct vertices such that $v_1v_2, v_2v_3, \dots, v_{n-1}v_n, v_nv_1$ are edges of G.

A graph is called a path if all its vertices and edges are those of a path in a graph. Similarly, a graph is a cycle if all its vertices and edges are those of a cycle.

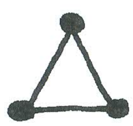
For $n \geq 1$, P_n denotes the path of order n, and for $n \geq 3$, C_n denotes the cycle of order n.

Examples :

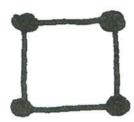
P_1 :  ← the trivial path

P_2 : 

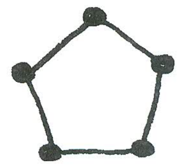
P_3 : 



$C_3 = K_3$



$C_4 = K_{2,2}$



C_5

Definitions : Two vertices u and v of the graph G is **connected** if there exists a u-v path in G. A graph G is connected if every two of its vertices are connected; otherwise it is **disconnected**.

Note that the relation "is connected to" is an equivalence relation on the vertex set of G. A subgraph of G induced by an equivalence class (of this equivalence relation) is called a **component** of G, that is, a component of G is a maximal connected subgraph of G.