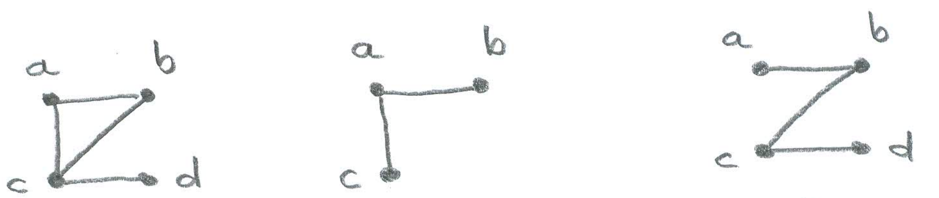
Definition: A graph H is a **subgraph** of the graph G, written $H \subseteq G$, if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$ such that, if $uv \in E(H)$, then $u, v \in V(H)$.
H is a **spanning** subgraph of G if $V(H) = V(G)$.

Example:



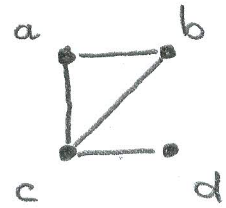
G H K

H and K are subgraphs of G. K is a spanning subgraph of G, while H is not.

Definition: If U is a non-empty subset of $V(G)$, then $\langle U \rangle$ denotes the subgraph of G **induced** by U, that is, $\langle U \rangle$ has vertex set U and two vertices of $\langle U \rangle$ are adjacent if and only if they are adjacent in G.

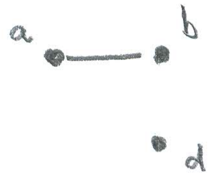
Similarly, if X is a non-empty subset of $E(G)$, then the subgraph $\langle X \rangle$ **induced** by X has edge set X and vertex set consisting of those vertices of G incident with the edges in X.

Example: Let G be the graph

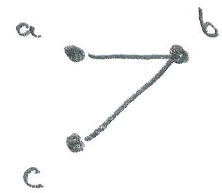


If $U = \langle a, b, d \rangle$ and $X = \{ab, bc\}$, then $\langle U \rangle$ and $\langle X \rangle$ are the graphs

$\langle U \rangle$:



$\langle X \rangle$:



Definitions of some classes of graphs:

- a graph is regular if all its vertices have the same degree; a r-regular graph is a graph G for which $\deg_G(v) = r$ for every $v \in V(G)$

- a graph G is a complete graph if every pair of vertices of G are adjacent, that is, if $|E(G)| = \binom{n}{2}$

- G is a bipartite graph if $V(G)$ has a partition into two subsets V_1 and V_2 such that every edge of G joins a vertex of V_1 to a vertex of V_2. In this case, V_1 and V_2 are called the partite sets of G