Groups of Permutations

Let G be a set of permutations of a finite set X. If G is a group, then we say that G is a group of permutations of X, and that G acts on X.

Example: All subgroups of S_3 are

$H_1 = \{id\}$ $\qquad$ $H_4 = \{id, (2\,3)\}$

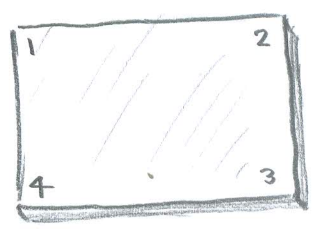
$H_2 = \{id, (1\,2)\}$ $\qquad$ $H_5 = \{id, (1\,2\,3), (1\,3\,2)\}$

$H_3 = \{id, (1\,3)\}$ $\qquad$ $H_6 = S_3$

They are all groups of permutations of $\{1,2,3\}$.

An example of a symmetry group of a geometrical object

Consider the rectangle with corners labelled 1, 2, 3, 4

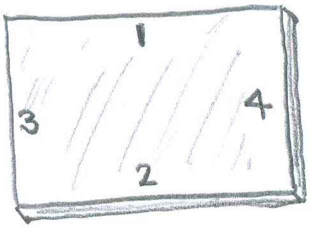


Let $G = \{id, (12)(34), (14)(23), (13)(24)\}$.

Then G is a group of permutations of the set $X = \{1,2,3,4\}$ of corners of the rectangle, that is, G acts on the corners of the rectangle. Note that G is a subgroup of S_4 and that the order of G divides the order of S_4, which is $4! = 24$.

We can also describe the symmetries of the rectangle as follows:

Consider the rectangle with sides labelled $1, 2, 3, 4$



Let $G = \{id, (12), (34), (12)(34)\}$.

Then G is a group of permutations of the set $X = \{1,2,3,4\}$ of sides of the rectangle, that is, G acts on the sides of the rectangle. Again, G is a subgroup of S_4 and the order of G divides the order of S_4.