Subgroups and Cosets

Definition: A subset H of a group G is a subgroup of G if H together with the binary operation of G forms a group.

Example: Consider the group $G_\Delta = \{i, r, s, x, y, z\}$ of the symmetries of the triangle.

$H_1 = \{r, s, z\}$ is not a subgroup of G_Δ, since it does not contain the identity element.

Is $H_2 = \{i, r, s\}$ a subgroup of G_Δ?

	i	r	s
i	i	r	s
r	r	s	i
s	s	i	r

The "multiplication" table of H_2 shows that we have closure. Of course we have associativity, since we have associativity in G_Δ. H_2 contains the identity element i, and from the "multiplication" table we can find the inverses of the elements of H_2 — i is its own inverse, whereas r and s are inverses of one another.

Therefore H_2 is indeed a subgroup of G_Δ.

The following theorem gives sufficient conditions for a subset to be a subgroup.

<u>Theorem 13.7</u> Let G be a group and H a non-empty subset of G. If the properties

S1 $x, y \in H \Rightarrow xy \in H$

and

S2 $x \in H \Rightarrow x^{-1} \in H$

hold, then H is a subgroup of G.

If G is finite, then S1 is sufficient; we don't have to check S2.

<u>Definition:</u> Let H be a subgroup of G. The <u>left coset</u> gH of H with respect to $g \in G$ is the set $\{gh : h \in H\}$.

<u>Example:</u> Consider $G_\Delta = \{i, r, s, x, y, z\}$ and its subgroup $H = \{i, x\}$.