TW 314 Lecture 2 (We 1 Feb. 2017)

A geometrical example of a group

Let Δ denote an equilateral triangle.

There are six transformations of Δ which leaves Δ in a fixed position in space.

These six transformations are called the six <u>symmetries</u> of Δ. They are...

i — do nothing

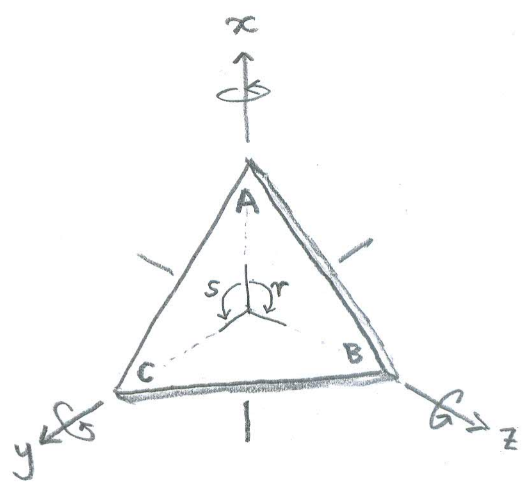
r — rotate clockwise through 120°

s — rotate clockwise through 240°

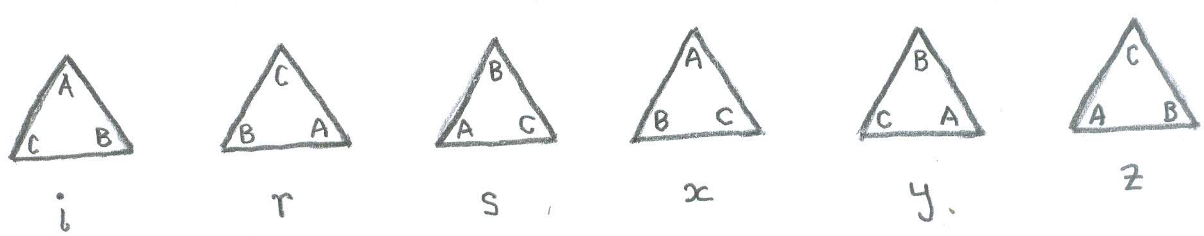
x — reflect in axis x

y — reflect in axis y

z — reflect in axis z



Initial position of the triangle Δ

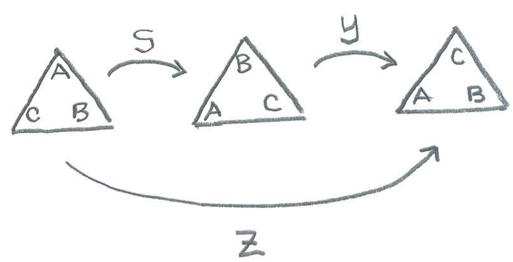


These are the effects of the symmetries applied on the initial position of Δ.

Let $G_\Delta = \{i, r, s, x, y, z\}$ and define the binary operation $*$ on G_Δ as follows:

$a*b$ is the symmetry transformation obtained by first doing the symmetry transformation b, and then doing the symmetry transformation a.

For example, $y*s = z$



Now $G_\Delta = \{i, r, s, x, y, z\}$ together with $*$ is a group: Clearly axioms G1 and G2 hold. Axiom G3 holds since i is the identity element. Also, each element of G_Δ has an inverse...

element:	i	r	s	x	y	z
inverse:	i	s	r	x	y	z

and therefore axiom G4 also holds.

$*$	i	r	s	x	y	z
i	i	r	s	x	y	z
r	r	s	i	y	z	x
s	s	i	r	z	x	y
x	x	z	y	i	s	r
y	y	x	z	r	i	s
z	z	y	x	s	r	i

← The group table for $(G_\Delta, *)$