Tutorial 1 Solutions

1. (a) The sum of real number are real numbers and addition of real number is associative. The identity element is 0 and the inverse (the negative) of a real number is a real number. So $(\mathbb{R}, +)$ is a group.

   (b) $(\mathbb{R}, \times)$ is not a group, since 0 does not have an inverse.

   (c) The product of nonzero real numbers are nonzero real numbers and multiplication of real numbers is associative. The identity element is 1, and the inverse (reciprocal) of a nonzero real number is a nonzero real number. So $(\mathbb{R} - \{0\}, \times)$ is a group.

2. The permutations of $\mathbb{N}_4$:

id	(12)	(132)	(1432)
(34)	(12)(34)	(1342)	(142)
(23)	(123)	(13)	(143)
(234)	(1234)	(134)	(14)
(243)	(1243)	(13)(42)	(1423)
(24)	(124)	(1324)	(14)(23)

3\. (a) $st = (123)(456)(78) \circ (1357)(26)$

$= (24587)(36)$

(b) $ts = (1357)(26) \circ (123)(456)(78)$

$= (16478)(25)$

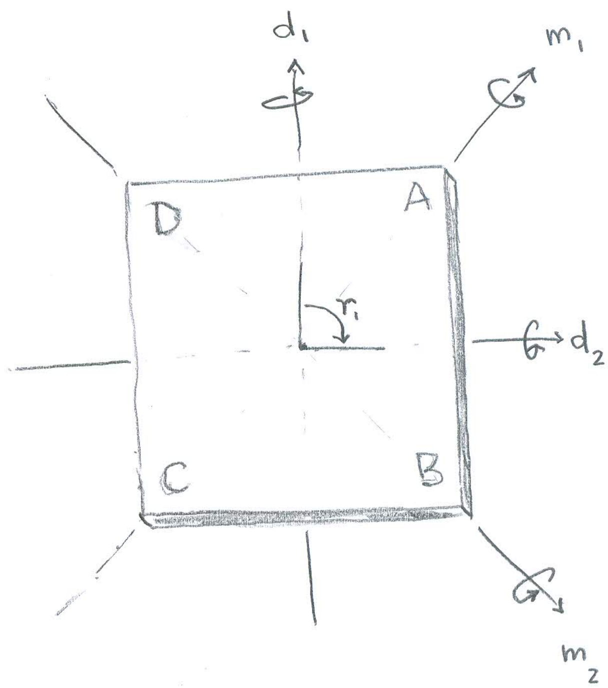
(c) $s^2 = (123)(456)(78) \circ (123)(456)(78)$

$= (132)(465)$

(d) $s^{-1} = (132)(465)(78)$

(e) $t^{-1} = (1753)(26)$

4\. The symmetries of a square:



i — do nothing

r_1 — rotate clockwise 90°

r_2 — rotate clockwise 180°

r_3 — rotate clockwise 270°

d_1 — reflect in axis d_1

d_2 — reflect in axis d_2

m_1 — reflect in axis m_1

m_2 — reflect in axis m_2

The group table is:

	i	r_1	r_2	r_3	d_1	d_2	m_1	m_2
i	i	r_1	r_2	r_3	d_1	d_2	m_1	m_2
r_1	r_1	r_2	r_3	i	m_1	m_2	d_2	d_1
r_2	r_2	r_3	i	r_1	d_2	d_1	m_2	m_1
r_3	r_3	i	r_1	r_2	m_2	m_1	d_1	d_2
d_1	d_1	m_2	d_2	m_1	i	r_2	r_3	r_1
d_2	d_2	m_1	d_1	m_2	r_2	i	r_1	r_3
m_1	m_1	d_1	m_2	d_2	r_3	r_1	i	r_2
m_2	m_2	d_2	m_1	d_1	r_1	r_3	r_2	i

5.

- $\alpha = (15)(27436)$

  $\alpha^2 = (24673)$

  $\alpha^3 = (15)(23764)$

  $\alpha^4 = (26347)$

  $\alpha^5 = (15)$

  $\alpha^6 = (27436)$

  $\alpha^7 = (15)(24674)$

  $\alpha^8 = (23764)$

  $\alpha^9 = (15)(26347)$

  $\alpha^{10} = id$

  The order of α is 10.

  Note that $\text{lcm}(2,5) = 10$

- $\beta = (1372)(46)$

  $\beta^2 = (17)(32)$

  $\beta^3 = (1273)(46)$

  $\beta^4 = id$

  The order of β is 4.

  Note that $\text{lcm}(2,4) = 4$

- $\alpha\beta = (163425)$ has order 6
- $\beta\alpha = (153476)$ has order 6