

8. exactly $24 - 20 = 4$.

3

(a)

9. $AB = S \Lambda S^{-1}$

Pre-multiply by B + Post-multiply by B^{-1}

$$BAB^{-1} = B S \Lambda S^{-1} B^{-1}$$

$$BA = B S \Lambda (BS)^{-1}$$

\uparrow \uparrow
e. vals e. vals

(b)

10.

$$A = \begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\lambda^2 - 5\lambda + 4 = 0 \quad (\lambda - 1)(\lambda - 4) = 0$$

$$x_k = C_1 (4)^k + C_2 (1)^k = C_1 4^k + C_2$$

(a)

11. $x'''' - 3x'' - 2x' - x = 0$

$$x'''' = 3x'' + 2x' + x$$

$$w' = 3v + 2u + x$$

$$u = x'$$

$$v = u' = x''$$

$$w = v' = u'' = x''''$$

$$w' = x'''''$$

4

$$x' = u$$

$$u' = v$$

$$v' = w$$

~~$$w' = 3v + 2u + x$$~~

$$w' = x + 2u + 3v$$

$$\begin{bmatrix} x' \\ u' \\ v' \\ w' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} x \\ u \\ v \\ w \end{bmatrix}$$

→ (e)

12. $\begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$

$$\lambda^2 - 10\lambda + 16 = 0$$

$$(\lambda - 8)(\lambda - 2) = 0$$

$$\sqrt{\frac{8}{8}} \text{ to } \sqrt{\frac{8}{2}} \text{ or } 1:2$$

→ (d)

13. $\begin{bmatrix} x & x' \\ x & y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ (b) or (c)

(b) $\frac{1}{25} \begin{bmatrix} 7 & -24 \\ 24 & 7 \end{bmatrix} \begin{bmatrix} 25 \\ d \end{bmatrix} = \begin{bmatrix} 7 \\ 24 \end{bmatrix}$

→ (b)

(c) $\frac{1}{25} \begin{bmatrix} -7 & 24 \\ 24 & 7 \end{bmatrix} \begin{bmatrix} 25 \\ d \end{bmatrix} = \begin{bmatrix} -7 \\ 24 \end{bmatrix}$