Department of Mathematical Sciences
Division of Applied Mathematics
Stellenbosch University

2019 | MS763 | Applied Markov Processes

Information

Course organiser: Prof Hugo Touchette
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Lectures: Tuesday, 11am-1pm in Eng A409
Tutorials: Friday, 10-11am in Eng A409

Assessment: 50% courseworks, 50% final test.

Description

Markov processes are used in many fields of science and engineering to model the evolution of dynamical systems perturbed by noise or manmade systems in which randomness or uncertainties play an important role. This module will give a basic introduction to Markov processes focused on their applications and simulation. The theory will cover Markov chains in discrete and continuous time, random walks, Brownian motion, stochastic differential equations, stochastic calculus, and large deviations. Applications will include word statistics, population dynamics, queues, particle systems, diffusions, noise-perturbed dynamical systems, stochastic control, and finance. The module will have tutorials to practice the theory and simulation methods (in Matlab or Python).

Outcomes

Upon completion of the module the student will be able to

- Understand the construction and properties of various Markov processes (Markov chains, jump processes, diffusions);
- Calculate probabilities and expectations related to Markov processes;
- Simulate the evolution of Markov processes and sample their statistics;
- Understand the range of applications of Markov processes;
- Formulate and use Markov models of physical and manmade systems evolving randomly in time.

Content

1. **Revision of probability theory** **Weeks 1-3**

   Axioms of probability theory; discrete and continuous random variables; conditional probabilities and Bayes's formula; independence; random variables; sums of random variables; law of large numbers; central limit theorem; generation of random variables; sampling methods.

2. **Discrete-time Markov chains** **Weeks 4-6**

   Markov chains, transition probability, probability evolution; classification of states; ergodic Markov chains, stationary distribution; reversibility; simulation and sampling methods; applications: random walks, queues, word statistics.

3. **Continuous-time Markov chains** **Weeks 7-8**

   Continuous-time limit, transition rates, escape rates; sample path properties; escape time; generator; adjoint generator; ergodic jump process; birth-death process; reversibility; simulation and sampling methods; applications: population dynamics, lasers, chemical reactions.

4. **Brownian motion** **Weeks 9-10**

   Random walks; definitions of Brownian motion; properties; generator; drifted Brownian motion; simulation; applications: diffusion, finance.

5. **Stochastic differential equations** **Weeks 11-13**

   Recap on differential equations; stochastic differential equations; basic examples; generator; Fokker-Planck equation; ergodic diffusions; stochastic calculus; Ito vs Stratonovich calculus; stationary distribution; current; gradient diffusions; simulation and sampling methods; applications: noisy dynamical systems, finance, physical diffusions, laser tweezers, controlled systems, large deviations.

Programming language

Matlab and Python will be used as programming languages for this module, but students can use other programming language or environment (C, Fortran, Mathematica, etc.).

References

- G. Grimmett and D. Stirzaker, *Probability and Random Processes*, 2001.
- K. Jacobs, *Stochastic Processes for Physicists: Understanding Noisy Systems*, 2010.
- B. Øksendal, *Stochastic Differential Equations*, 2000.

Other references will be given for each chapter. Lecture notes will not be provided as this is the first year the course is given.

Plagiarism and use of published code

- You must work independently on your courseworks.
- You are allowed to use any sources in your courseworks (books, internet, etc.), but you must cite them all.
- You are allowed to use published codes, but you must cite their source.
- Any suspected cases of copying or plagiarism will be referred to the department.