# Legendre-Fenchel transform of convex and nonconvex functions

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# Legendre and Fenchel

Adrien-Marie Legendre
1752-1833

No
(that's Louis)

Apparently, yes

$$f^*(k) = kx_k - f(x_k)$$

$$x_k : f'(x) = k$$

Werner Fenchel
1905-1988

$$f^*(k) = \sup_x \{kx - f(x)\}$$

Can. J. Math. 1(1) 73, 1949

# Two applications

## Large deviation theory

$$S_n = \frac{1}{n}\sum_{i=1}^{n} X_i, \qquad X_i \sim p(x), \text{ iid}$$

$$p(S_n = s) \asymp e^{-nI(s)}, \qquad I(s) = \sup_k \{ks - \ln\langle e^{kX}\rangle\}.$$

## Classical mechanics

Lagrangian mechanics

$$L(x, \dot{x})$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0$$

Hamiltonian mechanics

$$H(x, p) = p\dot{x} - L(x, \dot{x})$$

$$p = \frac{\partial L}{\partial \dot{x}}$$

$$\dot{x} = \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial x}$$

# Plan

- Convex sets
- Lower semi-continuity
- Subdifferentials
- Convex functions
- Legendre-Fenchel transforms
- Duality properties
- Generalizations

Some notes (click on the links):

Elements of convex analysis (HT)

Legendre-Fenchel transforms in a nutshell (HT)

A Course in Convex Analysis (A. Bossavit)