Complex networks: From Facebook to football

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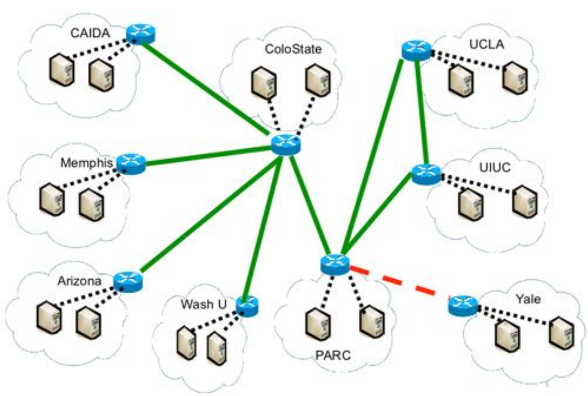
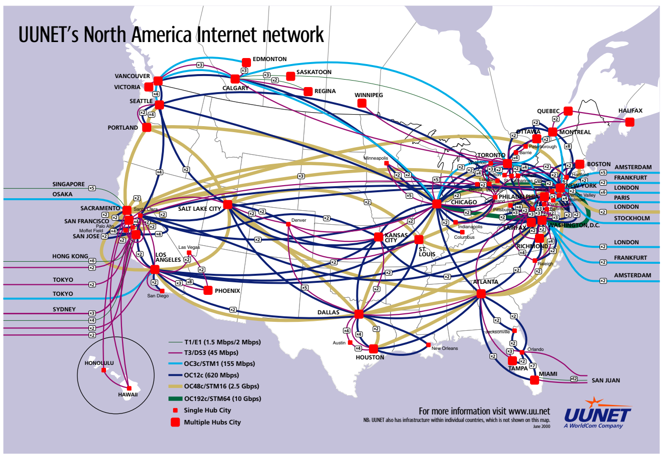
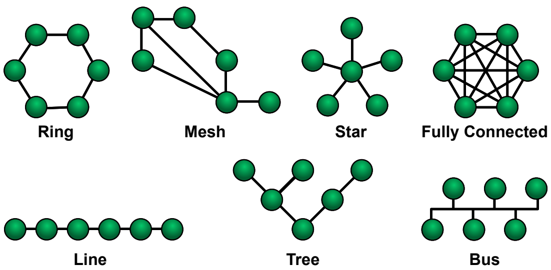
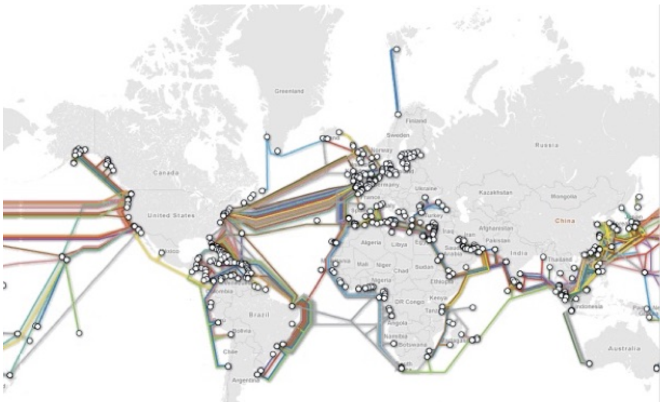
Internet



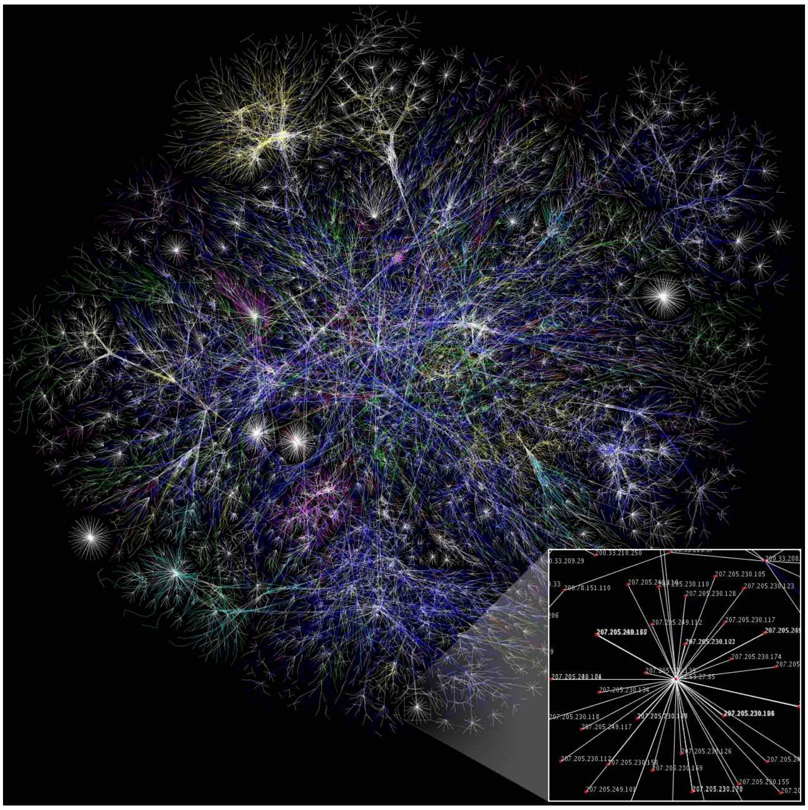




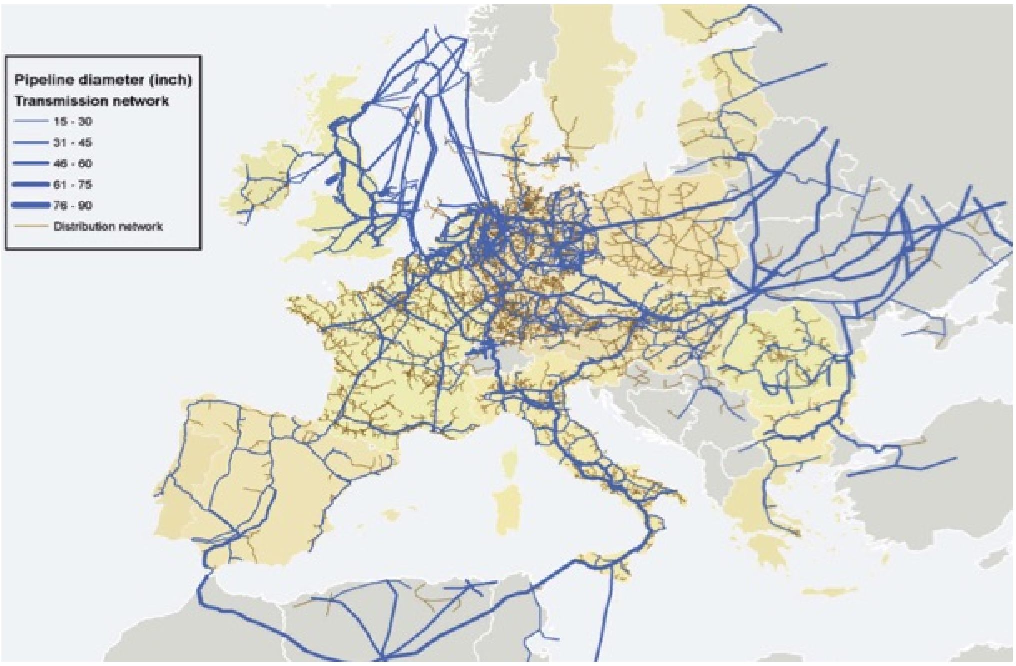


Internet (cont'd)

IP connection map, 15 Jan 2005 (from wiki)

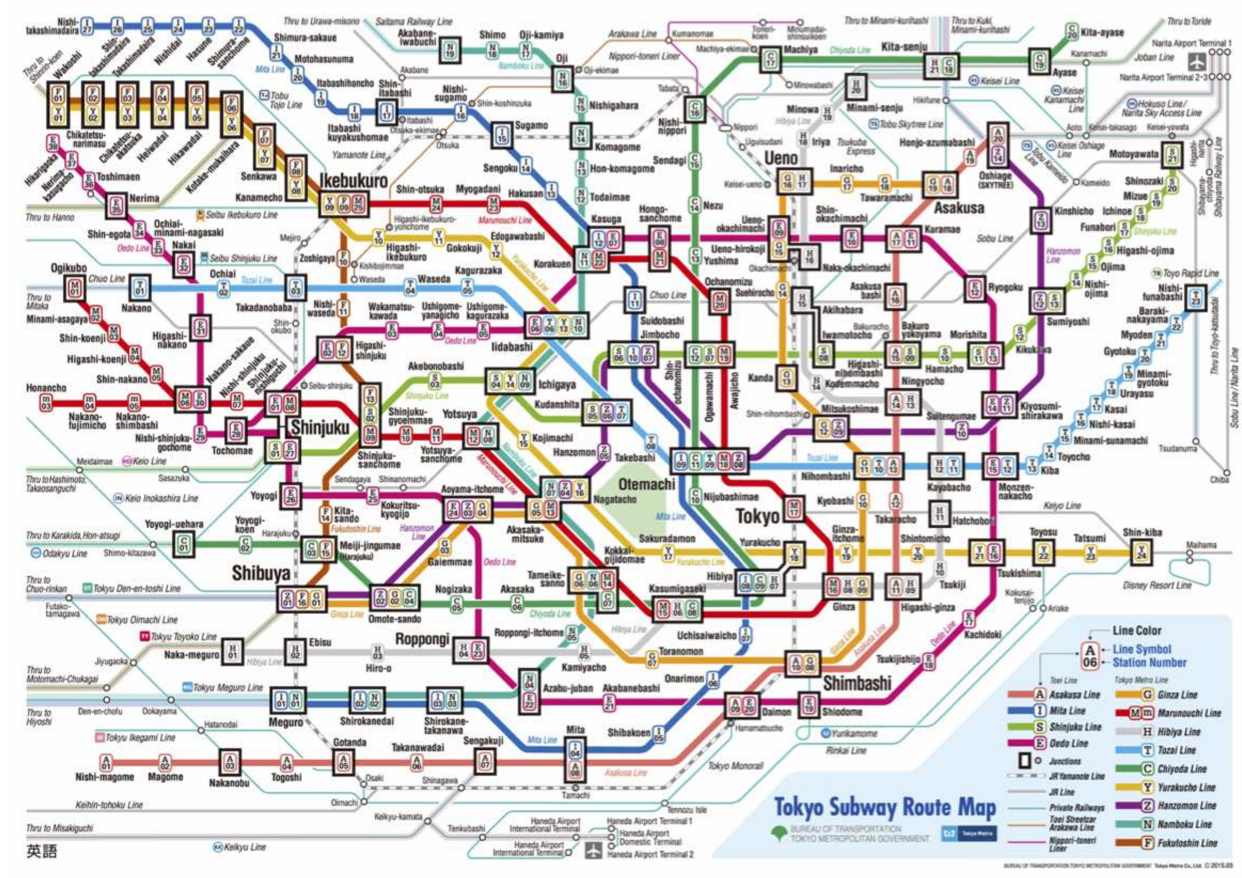


European gas network



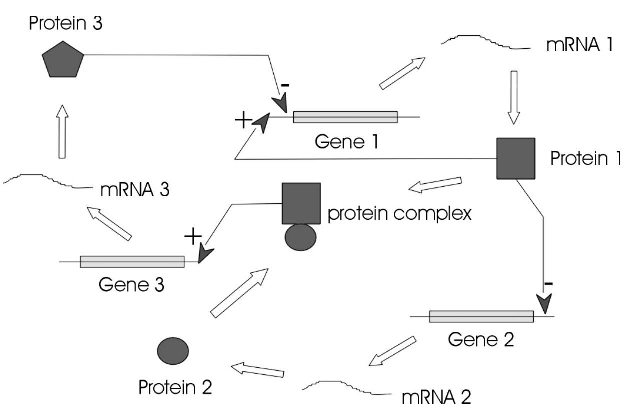
- $\sim$ 24 000 nodes, 25 000 links

Transportation

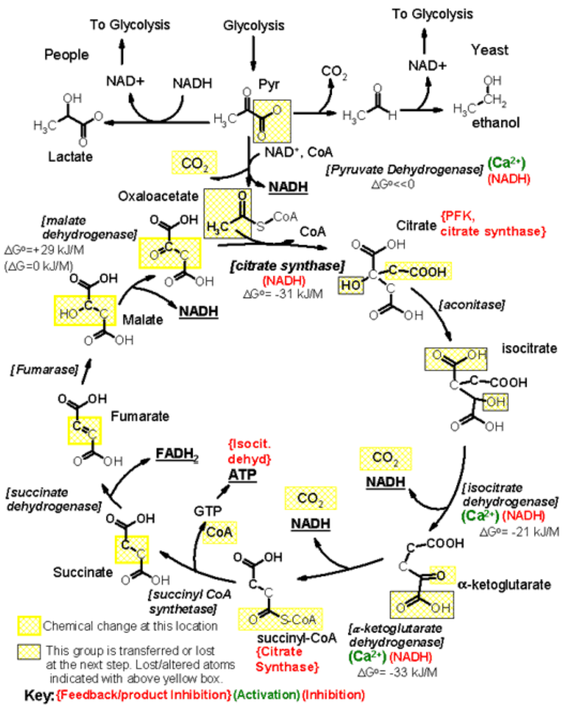


Biological and chemical networks

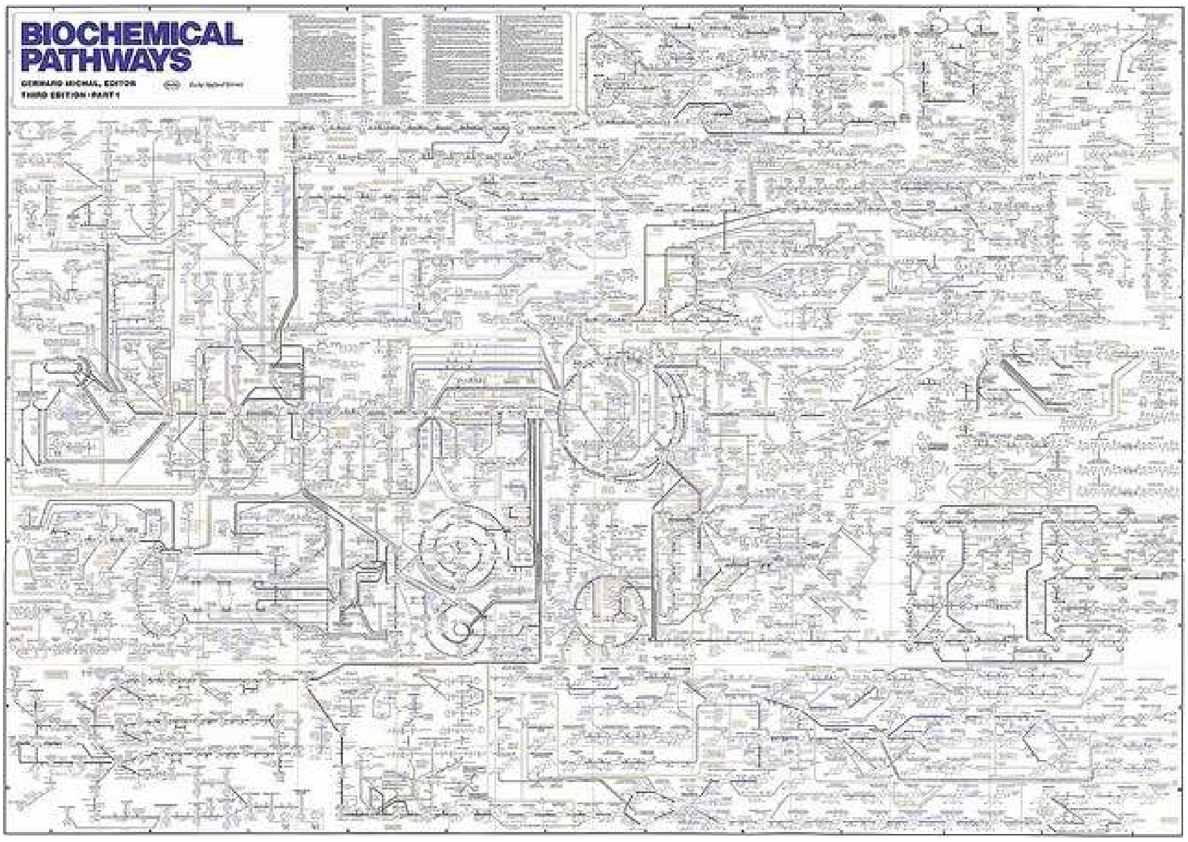
Gene regulatory network



Citric acid cycle

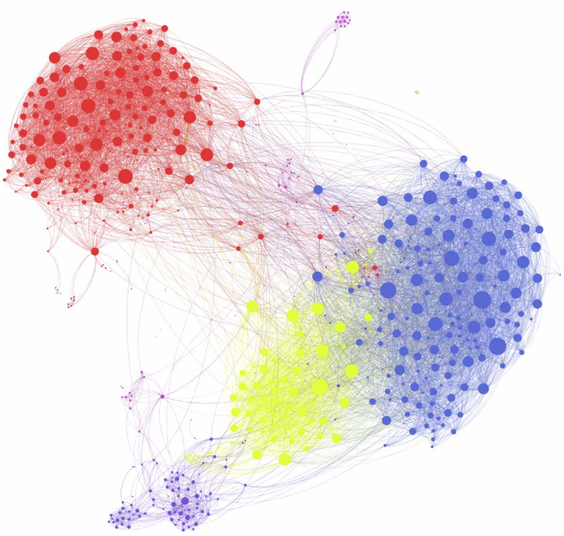


Biological and chemical networks (cont'd)

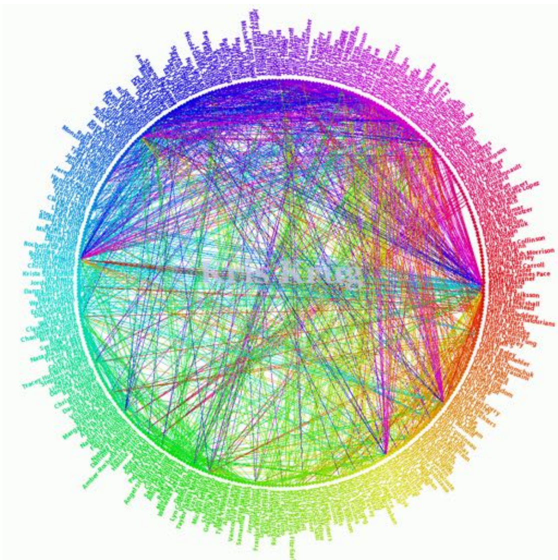


Facebook

Brendan Griffen



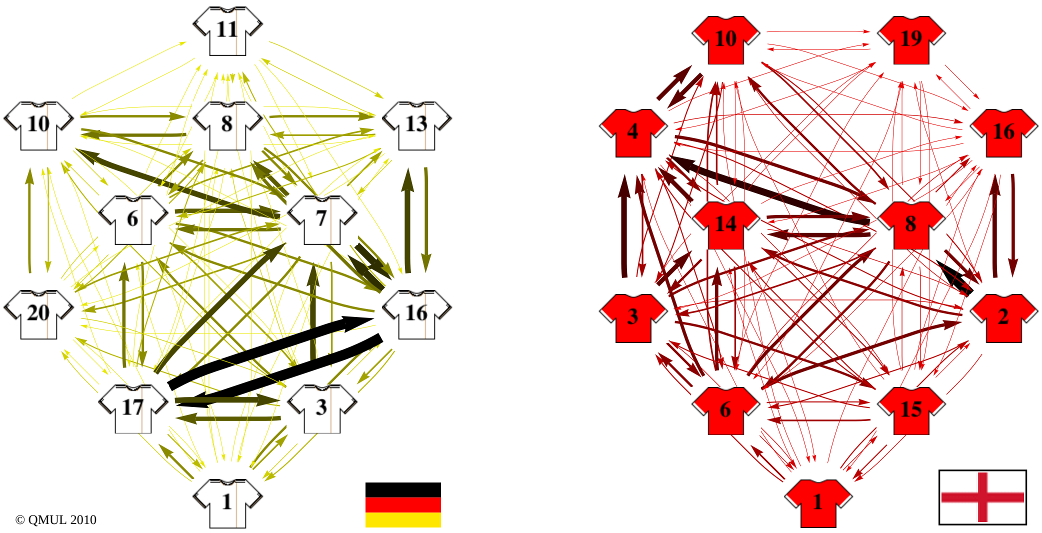
Kris Krüg



- Draw your Facebook network:
  https://msalganik.wordpress.com/2013/02/11/visualizing-your-facebook-network/
- More graphs:
  http://noduslabs.com/research/inclusive-exclusivity-innovative-networks/
  https://msalganik.wordpress.com/2013/02/24/a-gallery-of-personal-networks-from-facebook/

Football networks

2010 FIFA World Cup



Bits of graph theory

- $G = \{\text{nodes}, \text{edges}\}$
- Incidence/adjacency matrix:

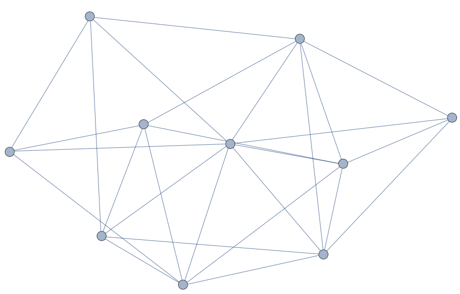
$$A_{ij} = \begin{cases} 1 & i,j \text{ linked} \\ 0 & \text{otherwise} \end{cases}$$

- Node degree:

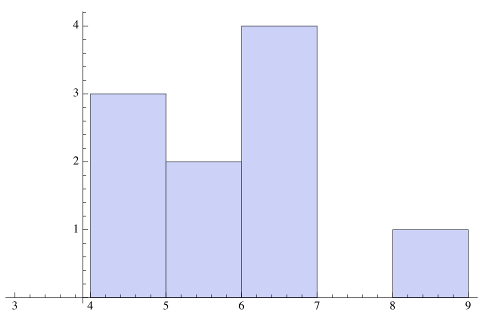
$$k_i = \sum_j A_{ij}$$

- Degree distribution: $P(k)$
- Shortest path (geodesic)
- Connected vs unconnected
- Largest component

10 nodes, 27 edges

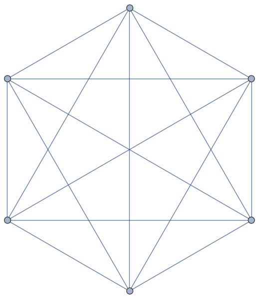


$\{4, 6, 6, 5, 4, 6, 4, 8, 5, 6\}$

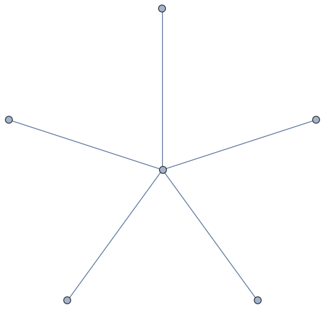


Regular graphs

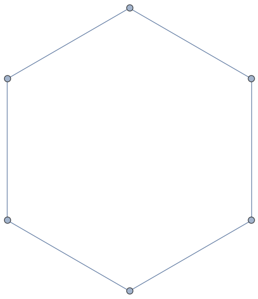
Complete



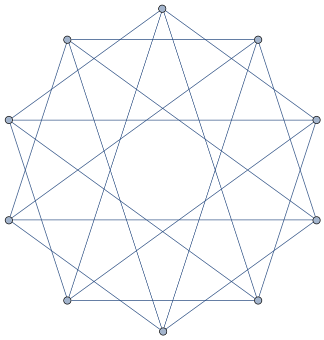
Star



Cycle



Circulant

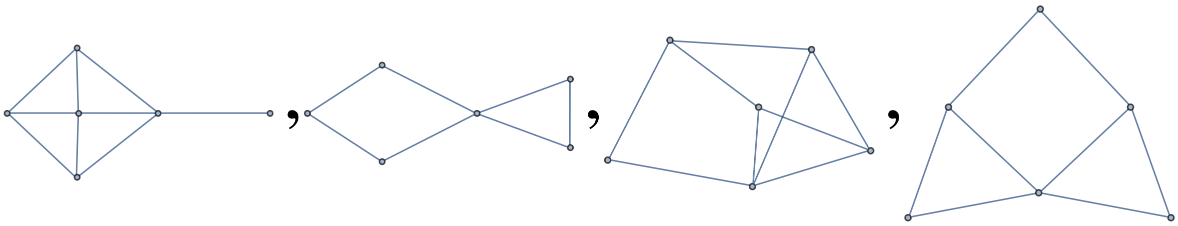


- Lattices
- ...

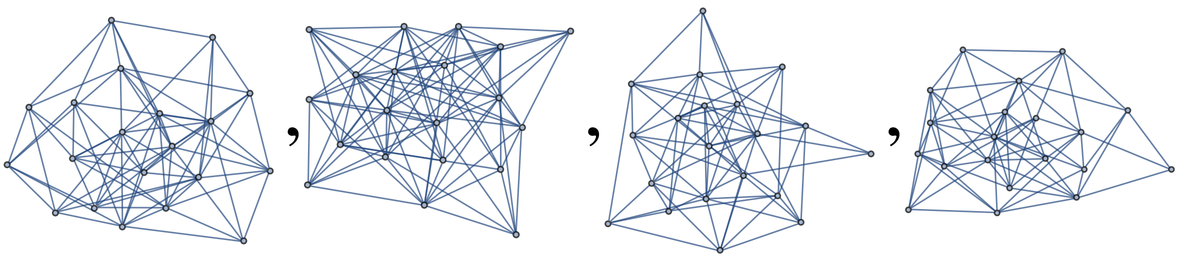
Random graphs

[Erdös & Rényi, 1959]

- N nodes
- Connect with probability p
- $(6, 0.5)$:



- $(20, 0.5)$:

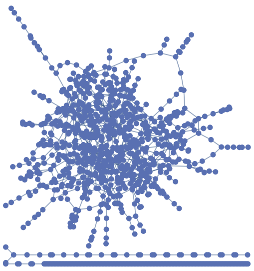
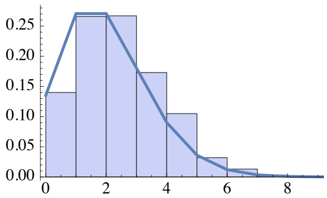


Random graphs (cont'd)

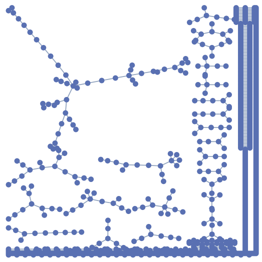
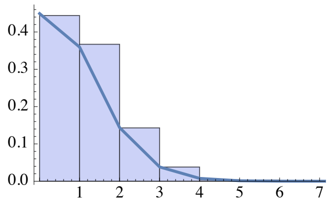
- Poisson degree distribution:

$$P(k) \to \frac{(Np)^k}{k!} e^{-Np}, \qquad N \to \infty, Np = \text{const}$$

- $Np < 1$: Likely disconnected
- $Np \geq 1$: Giant component
- Related to percolation transition

$N = 1000,\ Np = 2$

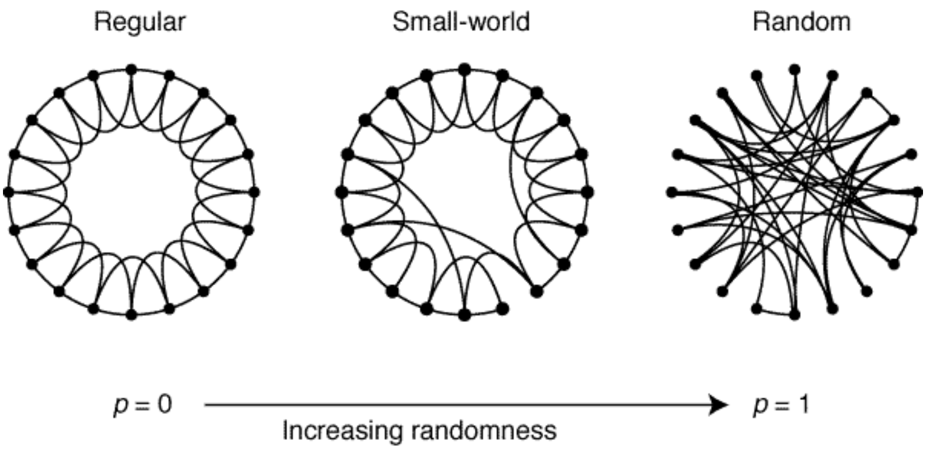



$N = 1000,\ Np = 0.8$

Small-world networks

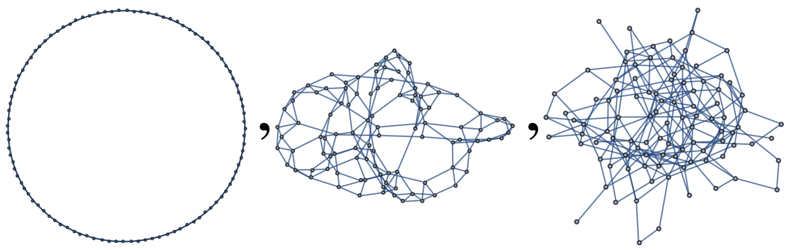
[Watts & Strogatz, 1998]

- Start with regular graph
- Rewire each edge at random with probability p
- Uniform re-attachment

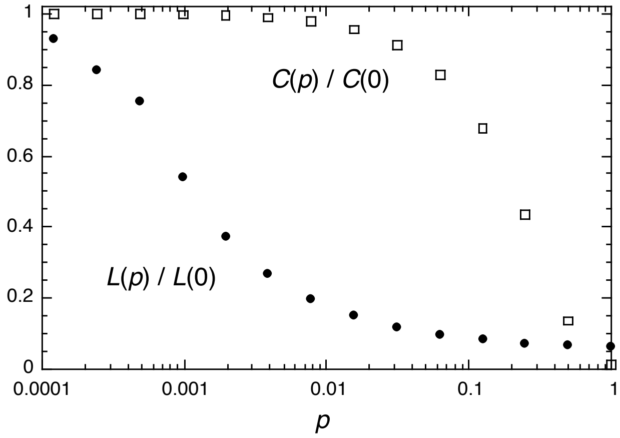


Small-world networks (cont'd)

- Efficient covering
- Small path length L
- High clustering C

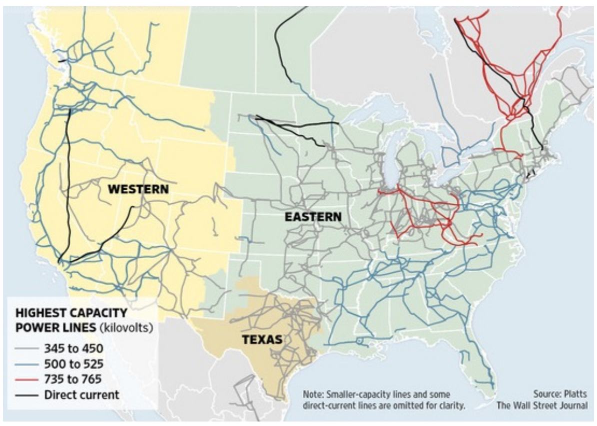


$N = 100,\ p = 0, 0.1, 1$



Applications

- Electricity grids
- Gene networks
- Social networks
- ...



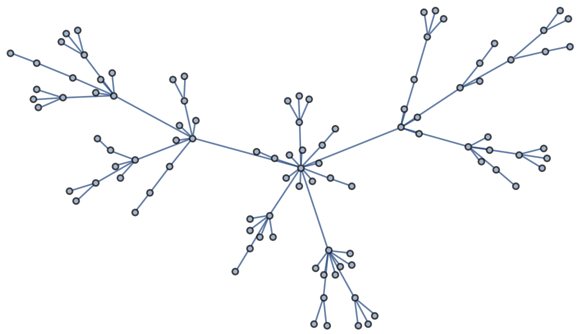
6 degrees of separation

- Kevin Bacon game (co-acting relation)
- No center: any actor/person will do
- Facebook app (average = 5.73)
- Erdös number
- Erdös-Bacon number (Sagan = 4 + 2, Feynman = 3 + 3)

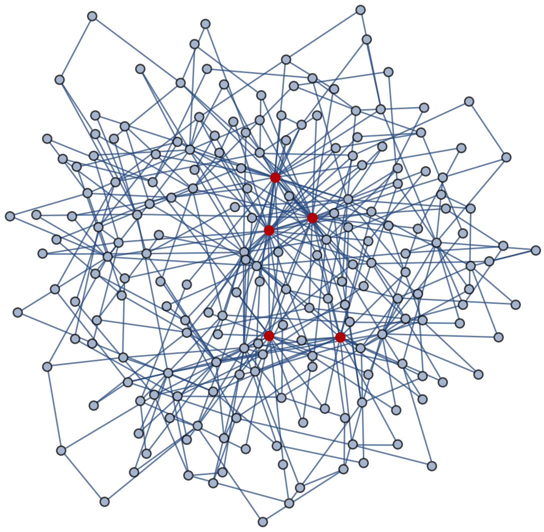
Scale-free networks

[Barabási & Albert, 1999]

- Start with n nodes
- Add new vertex with k edges
- Link edges to vertices at random in proportion to degree
- Preferential attachment



$N = 100, k = 1$



$N = 200, k = 2$

Scale-free networks (cont'd)

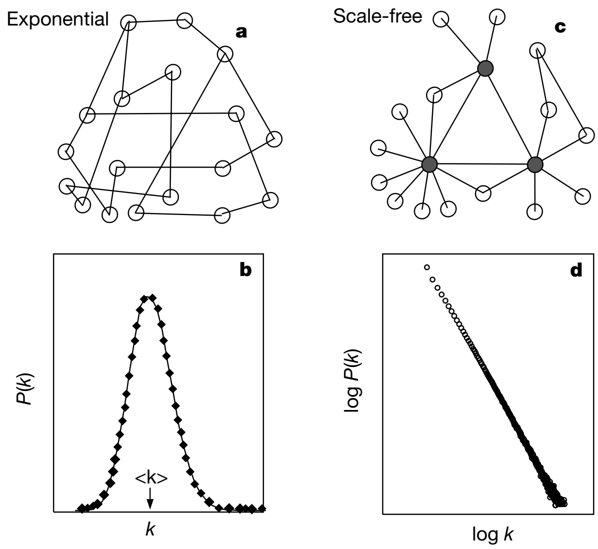
- Degree reinforcement: High degree nodes get more links
- Power-law degree distribution:

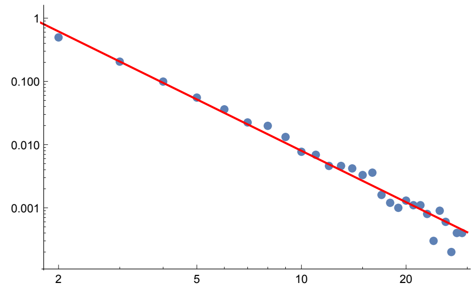
$$P(k) \sim \frac{1}{k^\gamma}, \qquad 2 < \gamma < 3$$

- Large hubs more likely

Examples

- Internet
- Scientific citations
- Neural networks
- Airline connections
- Stars?...

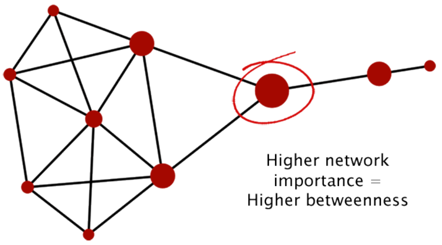


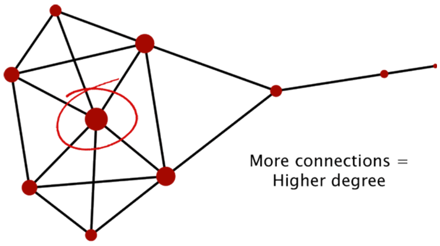


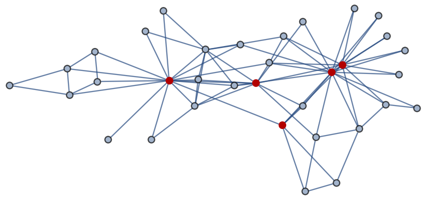
$N = 10^4, \ k = 2$

Hubs, centrality, and communities

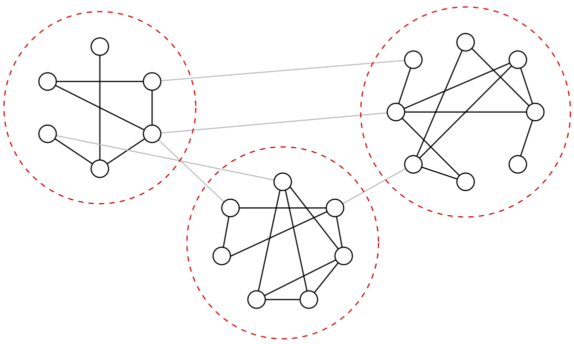
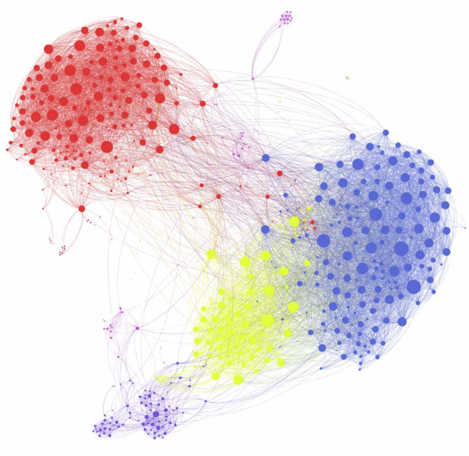
- **Hubs:** Central node, high centrality





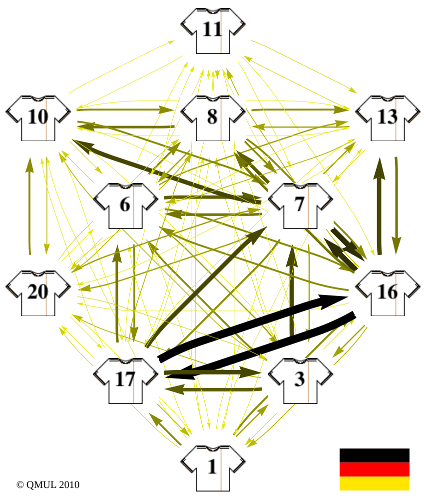


- Many definitions: Betweenness, degree centrality, pagerank, etc.
- **Communities:** Clusters, tight subgraphs

Football graphs

- With Javier López Peña
  (UCL, now at Kickdex London)
- 2010 FIFA World Cup in South Africa
- Passing data available (at the time)
- Data gathered with python script
- Pass networks generated with Mathematica
- Cumulative passes (all games added)
- Various centrality measures studied
- Predicted final result!
- www.maths.qmul.ac.uk/~ht/footballgraphs/
- Article: arxiv.org/abs/1206.6904



Measures

- Pass matrix: $A_{ij} = \#$ passes between players i, j
- Geodesic length: $d_{ij} =$ shortest (weighted) path between i, j

Closeness centrality

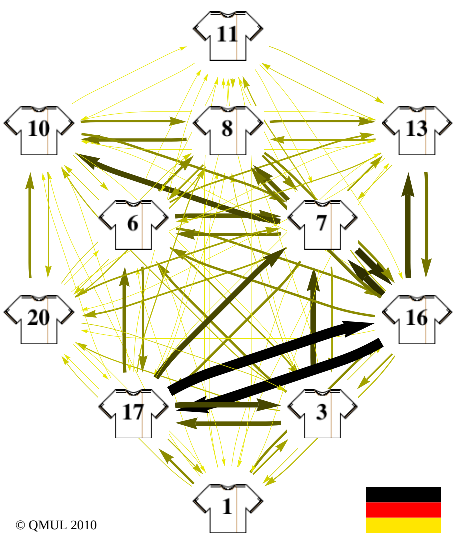
$$C_i = \frac{1}{\sum_{j \neq i} d_{ij} + \sum_{j \neq i} d_{ji}}$$

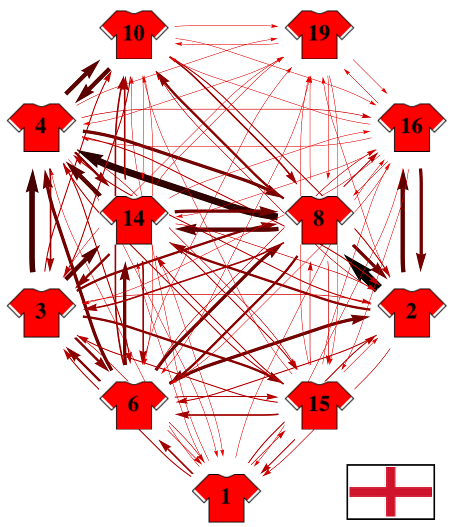
- High closeness = small distance
- Easily accessible player

Other quantities

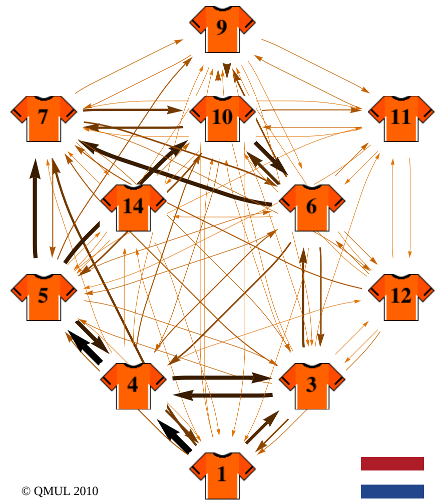
- Betweenness $C_B(i)$: Impact of removing a player (Hub related, 0 = player not involved)
- Pagerank x_i: Fraction of time player has the ball
- Clustering coefficient c_i^w: Clique factor

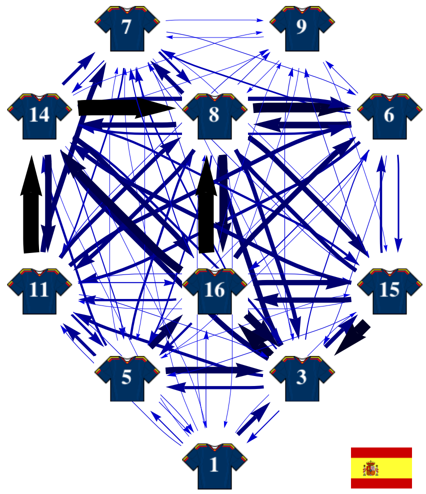
Germany vs England





The Netherlands vs Spain



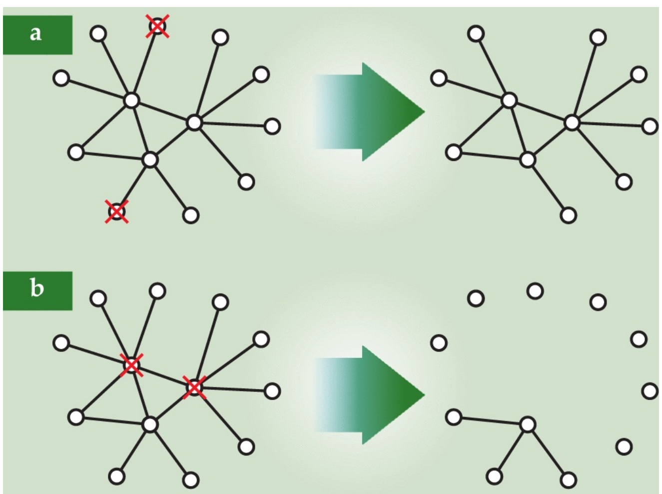


	C_i	$C_B(i)$	x_i	c_i^w
1 Stekelenburg	**16.34**	0.32	7.63	28.35
3 Heitinga	16.23	2.67	**11.06**	31.34
4 Mathijsen	**17.30**	1.30	10.84	33.22
5 V. Bronckhorst	15.74	1.12	10.07	**37.00**
6 Van Bommel	12.46	**3.08**	**11.19**	32.36
7 Kuyt	7.97	1.67	9.02	27.06
9 Van Persie	6.89	2.92	5.88	20.13
10 Sneijder	10.91	2.17	10.32	**33.77**
11 Robben	5.91	0.16	4.91	23.91

	C_i	$C_B(i)$	x_i	c_i^w
1 Casillas	16.52	0.00	3.29	20.46
3 Pique	17.32	**3.92**	11.46	30.70
5 Puyol	16.32	2.86	7.92	27.12
6 Iniesta	14.60	0.50	8.54	31.03
7 Villa	8.68	0.50	5.89	23.96
8 Xavi	**18.28**	1.19	**14.66**	**46.47**
11 Capdevila	16.54	**6.12**	10.56	29.91
14 Alonso	17.11	1.19	12.31	**41.69**
15 Ramos	16.45	2.41	9.02	27.05
16 Busquets	**18.55**	2.41	**12.99**	35.32

Research on complex networks

- Resilience/robustness of networks (random or targeted attacks)
- Immunology (centrality, communities)
- Layered networks (gas + electricity, distribution + consumption)
- Time-evolving networks (cities, transportation)
- Random walks on networks (Google, search, infections)



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